GENERAL ANALYSIS OF ALLOCATION OF FLOW INCREASES

by
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Chapter 1

INTRODUCTION

PURPOSE OF THE STUDY

The ultimate goal of this study is to analyze the effect on disposition of flow changes, for a hypothetical river basin, given variations in a series of water demand and supply characteristics. In order to derive the value of changes in flow at the point of origin, one must know the initial change in flow, predict what effects that initial change causes at each point of water use downstream of the initial change, and then estimate the value of each of those downstream changes.

This report focuses on the second of these items, the prediction of the effects of a flow change on users throughout a river basin. Further, the report focuses on a flow increase, such as that which may follow watershed vegetation changes (although the model has application to many types of flow changes). The present valuation of flow increases should indicate the proportion of any flow increases that would contribute to consumptive and nonconsumptive uses under a set of river basin features that characterize major basins of interest. This type of analysis requires a computer model that, by simulating the operation of the river basin, will help in the process of allocation of the water resource throughout the entire river basin. This report concentrates on the development of the computer optimization model to be used for that purpose.

GENERAL DESCRIPTION OF THE MODEL

Early studies of the value of flow increases often assumed that, once flow increases reached a major stream channel or reservoir, all or nearly all of the increases could be delivered to consumptive users. However, recent case studies have shown that several factors may severely limit the ability to deliver flow increases to users when needed. The timing of flows and flow increases, the capacity and management of reservoirs, and the timing of demand all affect the ability to deliver flow increases to consumptive users and the utility of flow increases in nonconsumptive uses.

The process of allocation of the water resource in the system, according to this study, will be dictated by an optimization model based exclusively on maximizing the sum of all economic benefits derived from the use of the water resource. That is, it will be only based on the economic value of water and not on the basis of a priority system. The multipurpose approach will identify

the trade-offs between the various water user groups. Each of the model components will be combined into a deterministic, multipurpose, multisite, river basin operational model.

The deterministic model to be developed will not explicitly consider uncertainty in the hydrologic variables. However, the stochastic nature of the inflows will be indirectly taken into account through an implicit stochastic optimization procedure. Many hydrologic traces will be synthetically generated and utilized as inputs to the optimization model.

The difference in total benefits realized from the system under two different inflow scenarios, with and without flow increases, will permit the analyst to assess the value of generating flow increases in the basin. This procedure, although optimistic when evaluating the attainable benefits due to the deterministic nature of the incoming inflows, will still be representative of realistic conditions of operation, since it is the difference between the two optimistic conditions that measures the value of the flow increases. Repetition of this procedure for several hydrologic traces will provide a distribution of the economic value of flow increases from which statistics can be computed.

The full scope of the study will be covered by three clearly differentiated study parts. The first part, **Flow Generation**, deals with all aspects related to the stochastic generation of flows. The synthetically generated flow traces will become the "natural" inflows into the reservoirs of the system. The second part, the **System Operation**, will define the optimal state of the system for any given operating scenario. And finally the third part, the **Operating Rules** module, will perform all the required statistical analysis of the gathered information. The operating rules will provide the set of guidelines that will help determine the allocation of flow increases under different types of operation. Figure 1.1 shows a simple flow-diagram of the three study parts. Detailed explanation of the first two study parts is provided in this report. The operating rules module is described briefly in the final chapter, and will be described in detail in a future report.

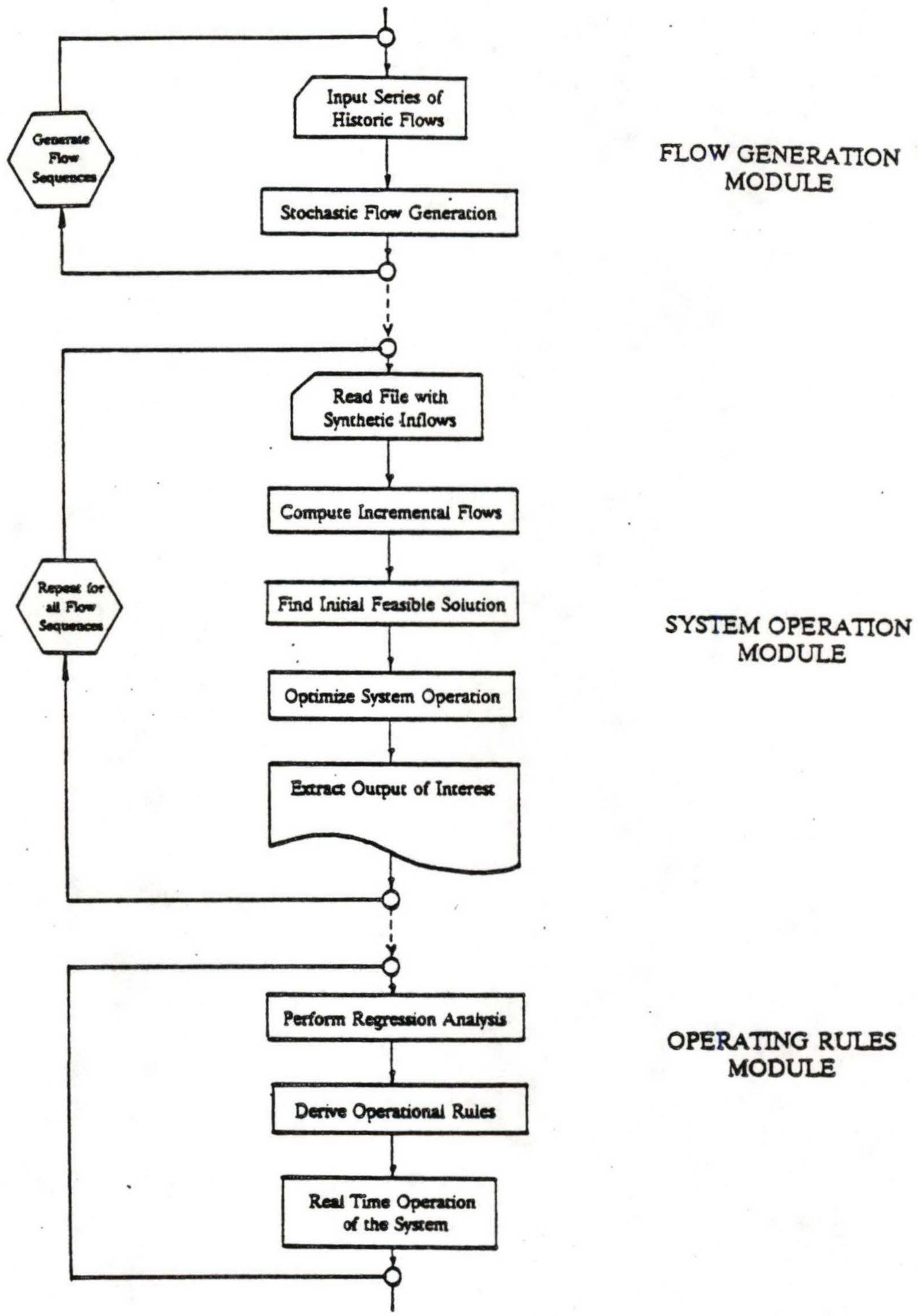

Fig 1.1 Flow-Diagram of the Study

CONFIGURATION OF THE HYPOTHETICAL RIVER BASIN

One of the first steps during the conceptualization of the model consists of defining the configuration of the hypothetical river basin to be modeled. According to the scope of the study, the topology of the system should be kept as simple as possible, while still considering the following basic requirements for the analysis of flow increases: (1) one or preferably two regions, in the upper parts of the basin, where artificial flow increases can be generated, (2) storage capacities which can potentially collect and regulate the natural and the artificially created flows, (3) river diversion points for withdrawing water to off-stream water users, and (4) a representative group of in-stream and off-stream water uses, namely: hydrogeneration and recreational water users among the first group, and irrigation, municipal and industrial water demands among the second group.

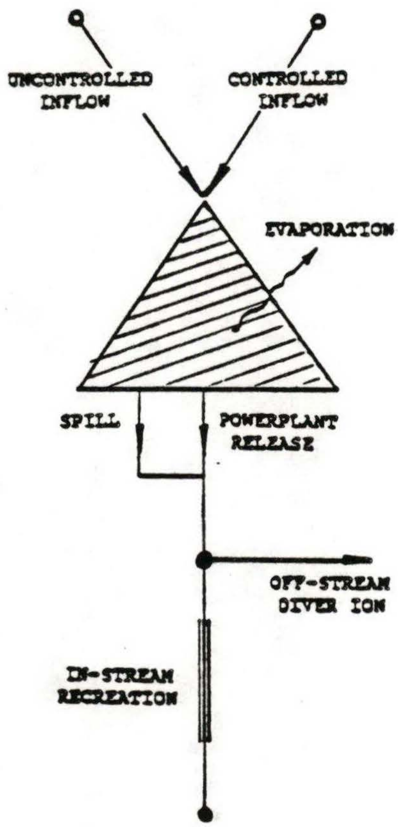

Fig 1.2 Basic River Subsystem

Before defining the configuration of the river basin, a so called "basic river subsystem" will be defined. Figure 1.2 depicts a single reservoir with all the inputs, outputs, and water users that can possibly be found in a system with the characteristics defined above. Two different type of inflows can enter the reservoir. The upper left arrow represents the natural or uncontrolled inflows, while the upper right arrow represents controlled inflows coming from upstream control structure/s. Evaporation water losses should also be accounted for in the model. They become particularly important during the analysis because of their volume being of the **same order of magnitude** as the flow increases. Besides evaporation, there are two other possible outputs from the reservoir: powerplant releases and spills. The latter encompasses two possible type of outputs, excess water drawn from the top of the pool by means of an uncontrolled (ungated) spillway, and controlled spills by means of a submerged pressure valve. Downstream from the impoundment there is a diversion point that draws water from the river to be allocated to a demand zone with consumptive use. Return flows from the

demand zone are not accounted for in the model. Finally, as also depicted in Figure 1.2, the most downstream portion of the subsystem is designated for water recreation purposes, as well as to guarantee satisfaction of downstream water rights.

The purpose behind the definition of this basic subsystem is two-fold: (1) to define a unitary component in such a way that any complex reservoir system can be assembled by linking together, in series and parallel, these basic units, and (2) to consider the basic unit not only as a functional constituent, but also as a "mathematical unit". This last consideration should facilitate the reformulation of the optimization model for any new reservoir system.

The system was originally conceived as the combination of three subsystems in parallel and series consecutively as depicted by the diagram in the left portion of Figure 1.3, where the

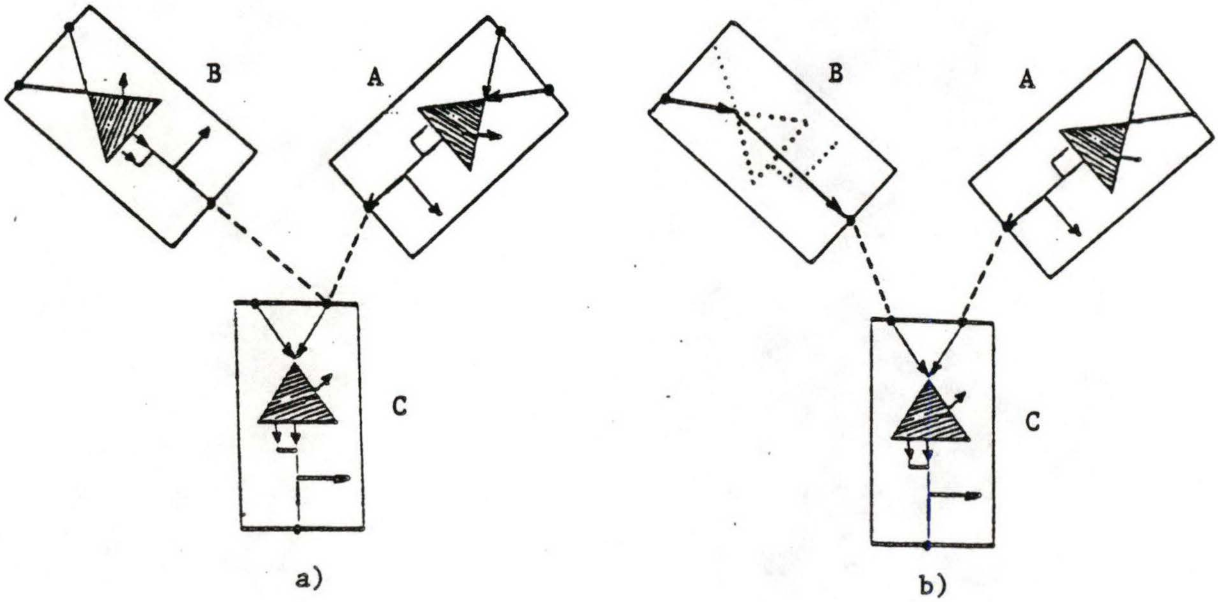

Fig 1.3 Linkage of Basic Subsystems, a) Initial Conceptualization b) Final System

subsystems are recognized by the letters A, B, and C. After some simplifications, the system was transformed from its original conception into a two reservoir system connected in series, as sketched in the right portion of Figure 1.3. Artificial flows can be generated in the upper portions of subsystems A and/or B. Subsystems A and C retain their full features as indicated in Figure 1.2, with a storage capacity, a powerplant, an off-stream demand zone, and a recreational reach in each subsystem. On the contrary, subsystem B has been highly simplified, by removal of the reservoir, so that it contributes only natural flows to the rest of the system. The configuration of the system

as finally adopted for this study is shown in Figure 1.4.

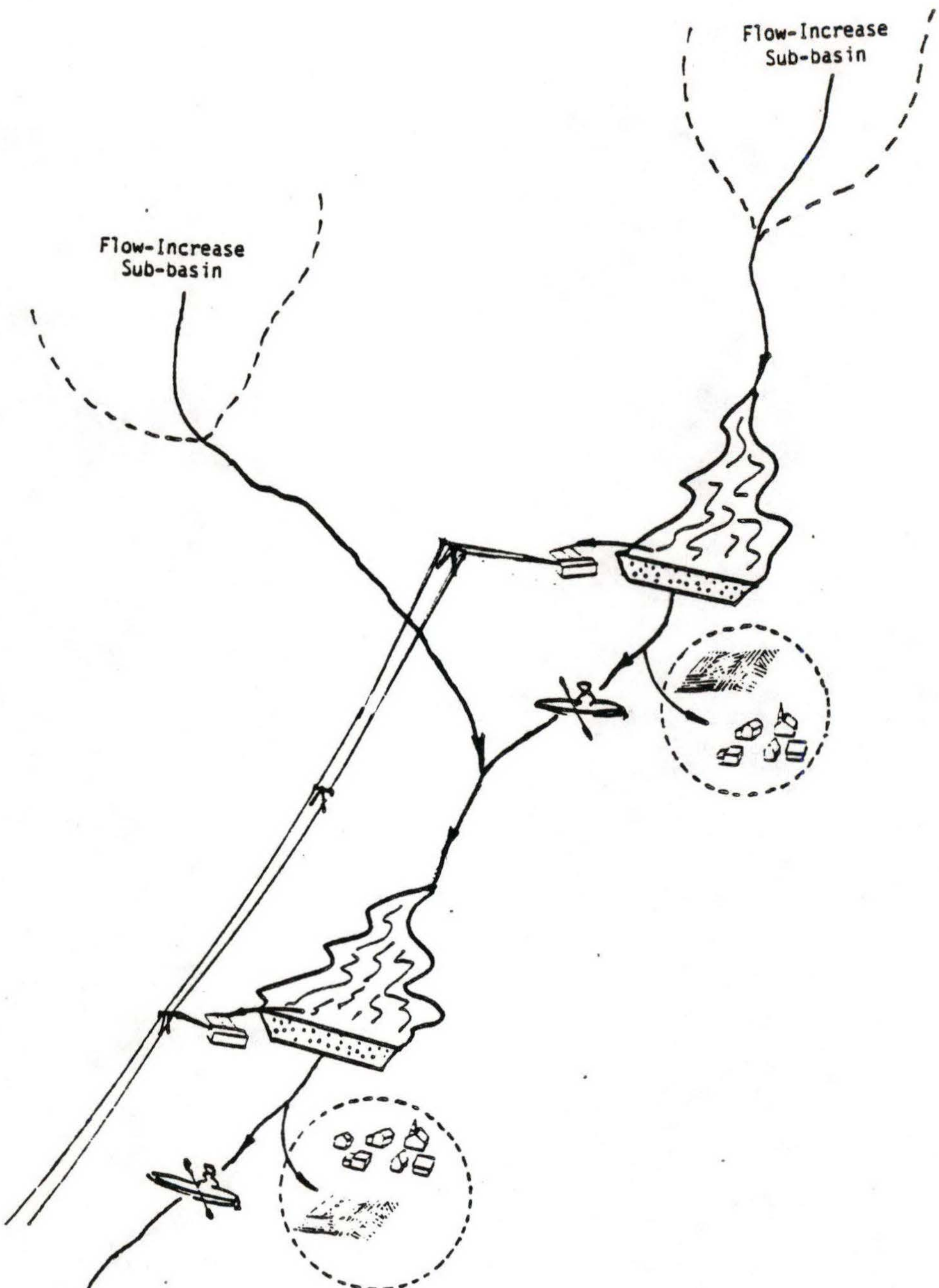

Fig 1.4 Hyphotetical River Basin

ORGANIZATION OF THE REPORT

The present report presents a comprehensive analysis of all major aspects concerning the development of the optimization model used to operate the hypothetical river basin. Each point of interest is discussed separately, introducing the adopted methodology in a comprehensive and complete manner. Furthermore, whenever applicable, application tests of the methodology are included.

Chapter 2 presents all the basic components of the optimization model. The objective functions associated with each water use are developed, as well as the operational constraints that constrict the operation of the system. By the end of the chapter, the method of solution for solving the present nonlinear programming problem is described.

Chapter 3 provides a description of a periodic autoregressive flow generation model to be used in conjunction with the optimization model. This chapter also summarizes the different approaches found in the literature for estimation of runoff increases to be added to the natural flows. At the end of the chapter, a description of the computational module that computes the stochastic flows is presented.

Chapter 4 describes the approach implemented for finding an initial feasible solution to the operational problem at hand, which is needed before starting the actual optimization process. A description and test of the computational module developed for this purpose is presented at the end of the chapter.

Chapter 5 is dedicated to the optimization algorithm used to solve the operational problem. The chapter presents the formulation of the nonlinear optimization problem within the framework of quadratic programming, including the set of constraints that guide the solution process. As in the previous chapter, a description and testing of the corresponding computational module is included.

Finally, Chapter 6 outlines the future implementation of the operational model, as well as its use for developing guidelines indicating disposition of flow increases of western hydrologic regimes.

Four appendixes are also part of this report. Appendixes I, II, and III are examples of the input files for running the computer modules I, II, and III respectively. Appendix IV provides a detailed derivation of all the partial derivatives utilized by the quadratic programming algorithm.

Chapter 2

COMPONENTS OF THE OPTIMIZATION MODEL

INTRODUCTION

Having described the purpose of the study and the topology of the water system in the previous chapter, we are now ready to introduce the basic elements that are part of the optimization model developed for solving the present operational problem. The chapter begins with a detailed introduction of the objective functions associated with the different water uses. Following this, the list of the operational constraints considered by the model is presented, as well as a discussion about the number and length of the time periods simulated by the model. The chapter concludes with a complete description of the method of solution implemented to solve the nonlinear optimization problem that results from the formulation of the problem.

STRUCTURE OF THE OBJECTIVE FUNCTION

Decisions for water allocation throughout the system and for the whole planning horizon are made on the basis of an objective which is to maximize the sum of all economic benefits from the instream and off-stream use of water. The following subsections begin with the introduction of the objective functions associated with hydroelectric power generation, off-stream water diversions and recreation. These components are then combined into a multipurpose objective function for the operation of the hypothetical river basin.

Revenues from Hydrogeneration

Hydroelectric energy production is a function of the head at the reservoir, the release through the turbines and the efficiency of the whole generating system. The main purpose behind this objective function is to fully exploit the economic opportunity to generate more efficiently (economic efficiency), given the available heads and flows in the reservoirs. The formulation of the hydropower objective function is mainly based on two concepts, the so called "Energy Rate" function and the "Marginal Price" function for energy. The energy rate function "$e(\bar{s})$" indicates the quantity of energy produced through the release of one unit of water (one MCM) through the turbines during a unit period of time (one hour) and for specific reservoir contents. By lumping the effects of turbine efficiency and reservoir head, the energy rate can be expressed as a function of only the reservoir storage. On the other hand, the marginal price function "$b(t)$", also known as the demand

curve for electricity, is the relationship that provides the economic return per Megawatt-hour sold as a function of the total volume released through the turbines during the time step used for simulating the operation of the system. As the total turbine release increases, the corresponding return per MWh sold (unit price) diminishes (law of diminishing returns). Figure 2.1 shows classical examples of these two functions.

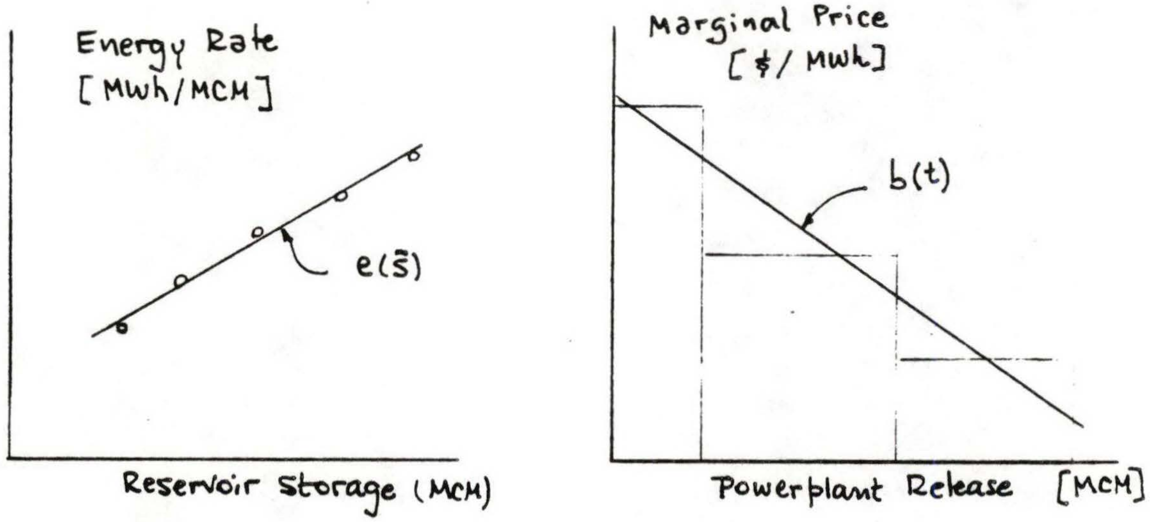

Fig 2.1 Energy Rate and Marginal Price Functions

Also, it is assumed that the hydropower system has no limitations as far as the amount of energy that can be supplied to the interconnected electrical network, where more than one type of energy is commercialized. For instance, if firm, secondary and dump energy are commercialized, they will be sold during on-peak, off-peak and night hours respectively. The marginal price function, as defined, implicitly includes the following assumptions: a) releases through the turbines are drawn at some constant rate, and b) water is released optimally, i.e., releases are done first during on-peak hours, then during off-peak hours, and whatever volume is left will be released during the night period. Details on how to build the marginal price function can be found in Laufer and Morel-Seytoux (1979) and Díaz (1988).

As indicated by Laufer and Morel-Seytoux (1979), the integration of $e(\bar{s})$ time $b(t)$ with respect to the turbine release yields the benefit stemming from hydrogeneration during a given time period, the i^{th} period, namely:

$$B_i^{POW}(T_i, \bar{S}_i) = \eta \int_0^{T_i} e(\bar{s}_i)\, b(z)\, dz \tag{2.1}$$

where T_i is the released volume through the turbines during period i, $\bar{S}_i$ is the mean reservoir content during the same period i, η is the generator efficiency, and z is a dummy variable of integration with the physical significance of released volume. The integration of (2.1) depends on the functional relationships representing the $e(\bar{s})$ and $b(t)$ functions. In this study, the energy rate and marginal price functions are represented by linear relations:

$$e_i\,[MWh/MCM] = a_1 + b_1\,\bar{S}_i \tag{2.2}$$

$$b_i\,[\$/MWh] = \frac{p_i}{P}\left(a_2 + b_2\,T_i\right) \tag{2.3}$$

where $\{a_1, b_1\}$ and $\{a_2, b_2\}$ are coefficients of the respective linear expressions. Again, $\bar{S}_i$ is the average reservoir content during the i^{th} period, and T_i is the turbine release (released volume) during that same time period. In Eq.(2.3), p_i is the price of energy in the i^{th} period of the year, whereas P is the average price of energy for the year. In turn, the average storage $\bar{S}_i$ in Eq.(2.2) depends on the history of inflows and outflows from the reservoir up to that period. Hence, $\bar{S}_i$ can be expressed approximately as:

$$\bar{S}_i = S_i^o + \frac{INF_i - OUT_i}{2} \tag{2.4}$$

where S_i^o is the reservoir storage at the beginning of period i, INF_i accounts for all inflow volumes to the reservoir during period i, and OUT_i represents all outflow volumes during that same period. Furthermore, OUT_i can be expressed as the sum of the turbine release T_i plus LOS_i, the latter encompassing all water losses and uncontrolled releases from the reservoir such as evaporation and spills,

$$OUT_i = T_i + LOS_i \tag{2.5}$$

Back substituting Eq.(2.5) into (2.4), and subsequently substituting Eq.(2.4) into (2.2), the expression for the energy rate function becomes:

$$e_i\left[MWh/MCM\right] = a_1 + b_1\left\{S_i^o + \frac{1}{2}INF_i - \frac{1}{2}T_i - \frac{1}{2}LOS_i\right\} \tag{2.6}$$

Now, substituting the final forms of $e(\bar{s})$ and $b(t)$, given by Eqs.(2.6) and (2.3) respectively, into the integral expressed by Eq.(2.1), the following is obtained:

$$B_i^{POW} = \eta \int_0^{T_i} \frac{p_i}{P}\left[a_2 + b_2 z_i\right]\left[a_1 + b_1\left(S_i^o + \frac{1}{2}INF_i - \frac{1}{2}z_i - \frac{1}{2}LOS_i\right)\right] dz \tag{2.7}$$

By further expanding the expression above,

$$\begin{aligned} B_i^{POW} = &+ \eta \frac{p_i}{P}\int_0^{T_i}\left[a_1a_2 + b_1a_2S_i^o + \frac{1}{2}b_1a_2INF_i - \frac{1}{2}b_1a_2LOS_i\right] dz \\ &+ \eta \frac{p_i}{P}\int_0^{T_i}\left[a_1b_2 - \frac{1}{2}b_1a_2 + b_1b_2S_i^o + \frac{1}{2}b_1b_2INF_i - \frac{1}{2}b_1b_2LOS_i\right] z\, dz \\ &+ \eta \frac{p_i}{P}\int_0^{T_i}\left[-\frac{1}{2}b_1b_2\right] z^2\, dz \end{aligned} \tag{2.8}$$

Finally, carrying out the integration and grouping the terms that have the same power of T_i, the economic benefit from the operation of a single powerplant during time period i is obtained:

$$\begin{aligned} B_i^{POW}[\$] = \eta \frac{p_i}{P}\Big\{ &T_i\left[a_1a_2 + b_1a_2S_i^o + \frac{b_1a_2}{2}INF_i - \frac{b_1a_2}{2}LOS_i\right] \\ &+ T_i^2\left[\frac{a_1b_2}{2} - \frac{b_1a_2}{4} + \frac{b_1b_2}{2}S_i^o + \frac{b_1b_2}{4}INF_i - \frac{b_1b_2}{4}LOS_i\right] \\ &- T_i^3\left[\frac{b_1b_2}{6}\right]\Big\} \qquad for\ i = 1,2,\ldots,N_p \end{aligned} \tag{2.9}$$

where all symbols have been previously defined except N_p which is the total number of periods. Notice that the objective function for hydrogeneration as expressed by Eq (2.9) turns out to be a third order function of the turbine release.

Revenues from Water-Supply

Off-stream water demand zones account for two main categories of water users, the "agricultural" user, including water for irrigation, livestock watering, etc, and the "urban" user, including municipal, industrial, and public use. Although historically urban and agricultural water have been supplied by separate projects, proper planning requires the coordinated provision of total water requirements.

Similarly to hydropower, the computation of revenues accruing from water diverted to a demand zone will also be based in the economic concept of marginal benefits. In other words, each water user will be represented by a demand curve (also marginal benefit curve) that it will indicate the maximum price that the user is willing to pay (or can afford to pay) to acquire an extra unit of water. Water usage will increase at lower prices and decrease at higher prices. Figure 2.2 displays typical marginal benefits curves for different off-stream water users. As expected, the demand curves reflect the different preferences of the different water users. While the municipal user is willing to pay high prices for only small amounts of water, farmers using water for irrigation are associated with a flatter curve, with low marginal prices for large allocations of water. The industrial user typically lies in between the other two users.

The optimal allocation of water in a multipurpose reservoir problem may combine several demand curves to obtain the so called aggregated or system demand curve. This approach, named Marginal Analysis, is a well known graphic optimization technique for the economic evaluation of multipurpose reservoirs (James and Lee, 1971). That same idea is used in this study to develop a combined demand curve that will represent all off-stream water users during each season. In this manner, even though each user individually may have different consumption preferences, if an aggregated demand curve can be defined, the total water diverted to a demand zone is considered as a lump sum of all uses.

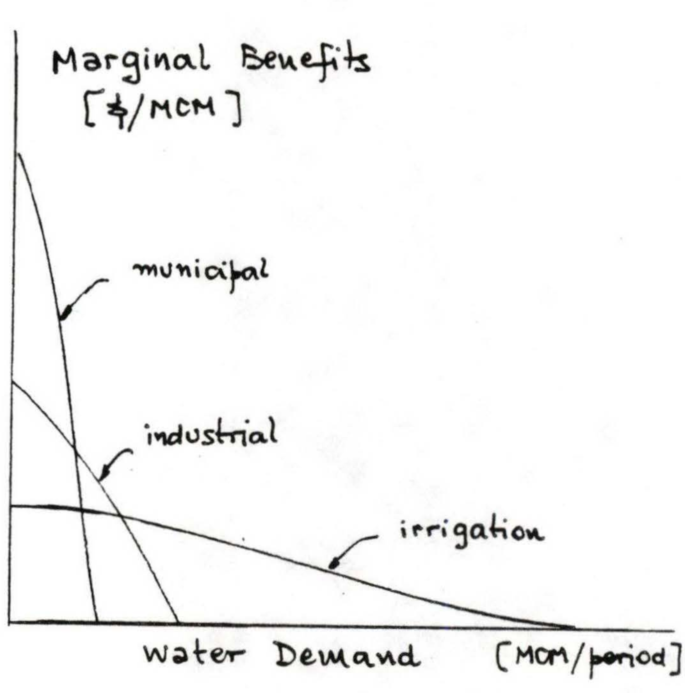

Fig 2.2 Demand Curves for Off-stream Water Users

Even though traditionally benefits for urban and irrigation water have been measured based on the annual allocation, this model computes off-stream benefits based on the monthly allocation of water. For that purpose, monthly aggregated demand curves must be defined. At the same time,

it is expected that by working with a set of demand curves (for instance, 12 curves per year), instead of a unique annual curve, the seasonal variations in demand, a characteristic of off-stream water use, will be automatically enforced in the optimization model by means of the water-supply objective function, making unnecessary the utilization of specific constraints to serve that purpose. In general, municipal water use is low in winter and high in summer. Industrial use is roughly constant during the entire year, while irrigation water distribution is entirely crop dependent. As an illustration, Figure 2.3 shows a typical monthly distribution of water as demanded by urban and agricultural users.

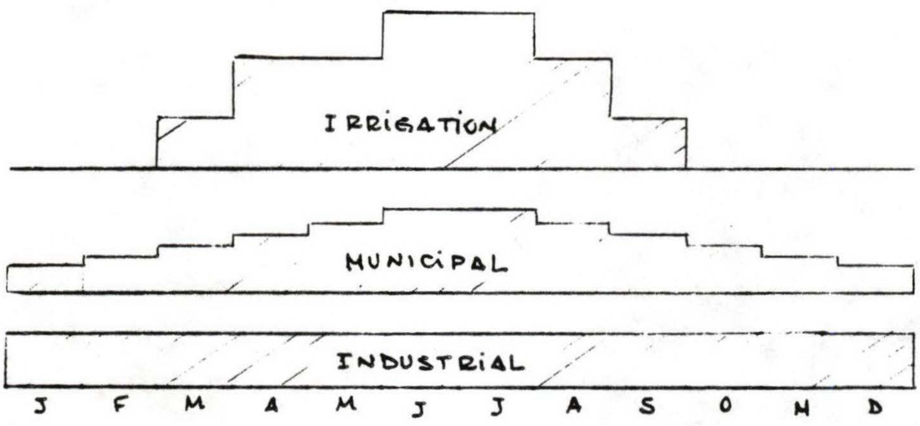

Fig 2.3 Monthly Distribution of Off-stream Water Demand

In order to combine marginal benefit curves, all curves should be marginal to the same variable. The multipurpose marginal benefit curve is obtained by summing horizontally the single purpose demand curves, as shown in Figure 2.4. In this manner, the water allocated to a demand zone during any given period can be considered as a lump sum for all users. For instance, during the month of january the aggregated curve will coincide with the urban demand curve for that month (no irrigation), whereas during june, it will result as the sum of the urban and irrigation demand curves that correspond to that particular month. If desired, once the optimal total water diversion for any given month is determined (by the optimization model), the total volume can be desegregated into the competing users by reversing the procedure outlined above. Division at the optimal marginal value gives the allocation per user, by making the marginal benefits to each use equal to each other.

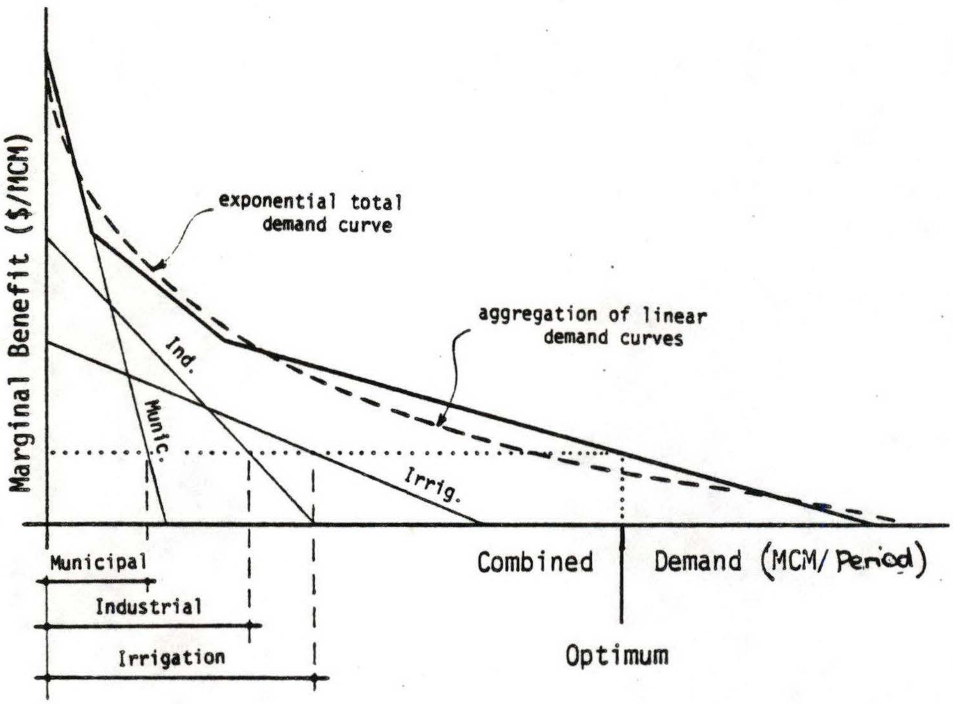

Fig 2.4 Aggregation of Demand Curves for all Water Users

The model assumes that the aggregated demand curves can be mathematically represented by exponentially decreasing functions of the water demand during the period,

$$b_i\,[\$] = a_3 \exp\left(b_3\, D_i\right) \tag{2.10}$$

where $\{a_3, b_3\}$ are coefficients of the exponential model, and D_i represents the aggregated demand during period i. Thus, the aggregated benefit for period i will be given by the following integral,

$$B_i^{WS}[\$] = \int_0^{D_i} a_3 \exp\left(b_3\, z\right) dz \tag{2.11}$$

After integration of the right side of Eq.(2.11), the benefit from the off-stream water diversion during period i has the expression:

$$B_i^{WS}[\$] = \frac{a_3}{b_3} \left\{\exp\left(b_3\, D_i\right) - 1\right\} \qquad for\ i = 1,2,\ldots,N_p \tag{2.12}$$

Notice that the objective function for water-supply as expressed by Eq.(2.12) turns out to be a nonlinear function of the monthly water diversion.

Recreation Purpose

Instream recreation considers uses such as fishing, boating, rafting, and any other activity without consumptive use of water. The present model formulation does not account for recreation benefits in a commensurable manner. It only imposes minimum flow requirements in the reaches downstream from the diversion points, in order to guarantee the existence of the recreational activities. (If so desired, economic benefits derived from recreation activities could be readily added to the model).

Total Revenues from the System

So far, benefits from each of the system components of the basin have been analyzed and the corresponding objective functions developed. At this point, it remains to combine all these components into a single objective function reflecting the multisite and multipurpose characteristics of the water system. Then, the overall objective will be to maximize the total return TR derived from all water uses generating revenues, and during the whole planning horizon. Mathematically, the total return is given by the double summation over all types of returns, and all periods:

$$TR[\$] = \sum_{i} \sum_{c} TR_i^c \tag{2.13}$$

for $i=1,2,...,N_p$, and $c=1,2,...,N_c$, where N_p and N_c represent the total number of time periods and components of the system respectively. The multiobjective function (2.13) will identify the optimal trade-offs between the various competing water users, basing the allocation of water only on its economic value.

OPERATIONAL CONSTRAINTS

The strategy to be followed in the operation of the hypothetical reservoir system is the one that maximizes the total return over a planning horizon, subject to several operational constraints. These constraints impose physical limits in the operation of the reservoirs and other parts of the system. This section presents a list containing those constraints that are considered essential for the proper simulation of the system. But before describing the operational constraints, let's define all the variables that can possibly be found in any subsystem of the river basin as described in Chapter 1. Figure 2.5 shows a scheme of a three reservoir system in series, displaying all the inputs and outputs that the model considers. During the model formulation, these inputs and outputs become the links necessary to mathematically connect a particular subsystem (for instance the central reservoir) with other subsystems, like the ones located upstream and downstream from it. The basic notation is as follows,

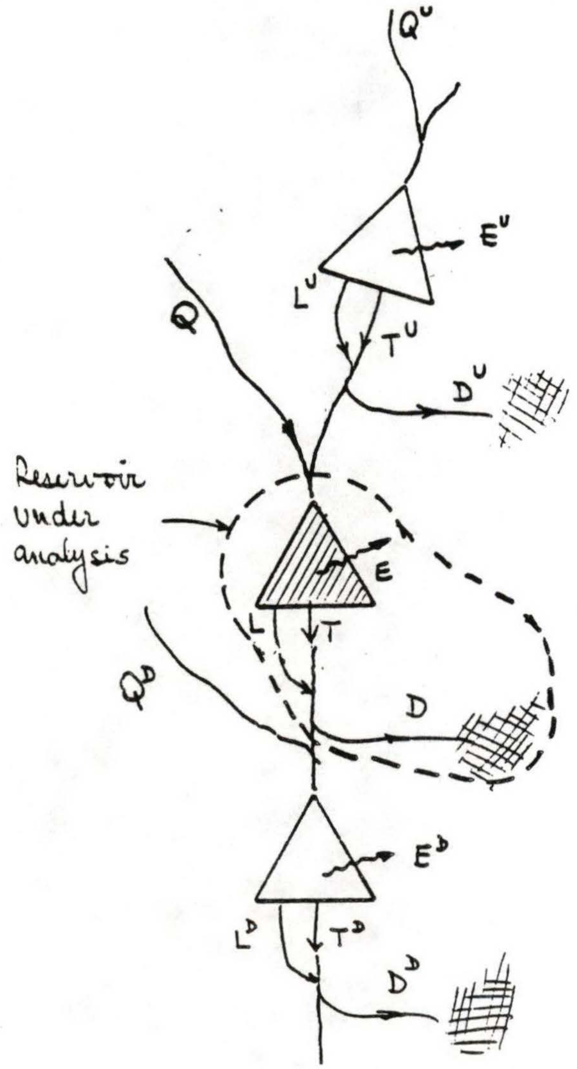

Fig 2.5 Reservoir System - Notation

T	turbine release
D	off-stream diversion
L	reservoir spill
E	net evaporation
S	reservoir storage volume
Q	unregulated inflow

and where the meaning of the superscripts [^{u}, ^{d}] is explained below. Some other notation of interest is,

S^o	initial active storage volume
S^f	final active storage volume
S_M	maximum operational storage
S_m	minimum operational storage
Frec	Recreational flow

Given the multi-site and multi-purpose characteristics of the model, the notation necessary for writing the equations becomes more complex. It is necessary to clearly distinguish between variables associated with different water users as well as different portions of the system. The notation adopted in this study is:

(i) there is a unique letter associated with a particular water use. For instance, T stands for turbine release, regardless of the powerplant the release is from,

(ii) the subscript i is the index for time period identification. For instance, i=3 denotes the third month in the optimization horizon,

(iii) the superscripts [^{u}, ^{d}] are used to differentiate a variable as belonging to a subsystem located upstream or downstream respectively from the subsystem under consideration.

Referring back to Figure 2.5, total inflows and outflows from the reservoir at a given time period (i) can be expressed as,

$$INF_i = Q_i + T_i^u + L_i^u - D_i^u \tag{2.14a}$$

$$OUT_i = T_i + L_i + E_i \tag{2.14b}$$

The extent to which an optimization model needs to represent the actual operation of a complex water system gets reflected in the size of the constrain set. A system with a complex operation will demand a very large constraint set, whose size will depend on the number of system components and the total number of time periods simulated by the model. A list of the operational constraints that are considered essential for the purpose of the study is:

Capacity Constraints: These constraints are required to ensure that the storage at the end of any period S_i does not exceed the maximum reservoir capacity. These constraints apply for both reservoirs, A and C (see Figure 1.3),

$$S_i = S^\circ + \sum_{k=1}^{i} (Q_k + T_k^u + L_k^u - D_k^u) - \sum_{k=1}^{i} (T_k + L_k + E_k) \leq S_M \tag{2.15}$$

Release Constraints: They are required to ensure that the reservoir releases at any period i will not deplete the reservoir storage beyond the minimum active capacity. These constraints apply for both reservoirs, A and C,

$$S_i = S^o + \sum_{k=1}^{i} (Q_k + T_k^u + L_k^u - D_k^u) - \sum_{k=1}^{i} (T_k + L_k + E_k) \geq S_m \tag{2.16}$$

Prescribed Final Storage Constraints: are required to prevent the optimization algorithm from generating extra benefits at the expense of depleting the reservoirs by the end of the simulation cycle. These constraints apply for both reservoirs, A and C,

$$S^o + \sum_{k=1}^{N_p} (Q_k + T_k^u + L_k^u - D_k^u) = \sum_{k=1}^{N_p} (T_k + L_k + E_k) + S^f \tag{2.17}$$

Powerplant Capacity Constraints: are required to limit the maximum and minimum discharge through the turbines during any period i. These constraints apply for both powerplants, A and C,

$$T_m \leq T_i \leq T_M \tag{2.18}$$

Water-Supply Constraints: are required to limit the maximum and minimum off-stream diversion at any period i. These constraints apply for both diversion zones, A and C,

$$D_m \leq D_i \leq D_M \tag{2.19}$$

Minimum Recreational Flow Constraints: are required to ensure instream-flow maintenance to support aquatic resources during any period i. These constraints apply for both reaches downstream from the off-stream diversion points,

$$T_i + L_i - D_i \geq Frec_i \tag{2.20}$$

The previous list indicates that even a modest constraint set will include hundreds of equations. Besides the constraints listed above, there are some other constraints that might also restrict the operation of the system. What follows is an enumeration of these other constraints, and a brief explanation of why they were not included above. For instance: Maximum energy sale: because it is assumed that the system can sell all the energy that is able to generate,
Maximum spillway capacity: because of the large time step adopted, monthly periods, the regulation effect of the spillway becomes irrelevant; Seasonality of water-demand: because the specification of different marginal price curves for different water-demand periods should account for the seasonal variations in demand, Diversions not to exceed reservoir releases: because this requirement is automatically satisfied when minimum recreational flows are enforced.

NUMBER AND DURATION OF TIME PERIODS

Given the purpose of this study, the decision on the number of time periods to simulate in the model and the length of each period is a critical one. If the reservoir system is to contain enough storage for managing water between years, then a large number of periods encompassing several years of operation must be included.

In this manner, benefits of storing excess water during wet years for future releases during dry years can be realized. Furthermore, if both ends of each optimized trace have to be disregarded in order to remove the effect of imposed boundary conditions, an even larger number of time periods would be required. But, on the other hand, the longer the optimization horizon, the larger the size of the Quadratic programming (QP) problem to be solved. Execution time and memory requirements then become a concern, particularly if a microcomputer is to be used for carrying out the computations. For all these reasons, the number and duration of time periods should be the result of a trade-off between the objectives of the study and the computer resources available. In this study, monthly time periods were adopted. Furthermore, the model was coded using PARAMETER statements for array declarations, DO-loops, names for constants, etc; which not only makes the program more readable, but also provides a way to easily modify the code to meet new requirements simply by changing the values of certain constants listed in parameter statements. Initially, the model has been set up for two years of monthly operations. Considering four different water users, it requires a total of 96 control variables. However, the increase in size of the QP formulation relates not only to the number of control variables, but with the size of the constraint set as well. At this point, the relationship between computational load versus the size of the QP formulation is unknown, but it will certainly not be linear. It is suggested that once this initial problem size is found manageable by the selected computer system, then to start increasing the size of the problem gradually until the desired simulation horizon is satisfactorily handled.

METHOD OF SOLUTION

Nonlinear programming problems are commonly encountered in many water resources applications. There are a variety of approaches in dealing with such problems, but none of which is uniquely superior or universally proven. A special case of the general nonlinear programming problem takes place when the objective function is reduced to a quadratic form, and all the constraints are linear. Although a quadratic function is the simplest nonlinear approximation that can be used for a nonlinear objective, it has been shown to be particularly suitable to solve the

proposed reservoir system operation problem (Díaz and Fontane, 1989).

Definition of the Problem

The problem is to maximize a general nonlinear, nonquadratic objective function subject to linear constraints,

$$\underset{x}{Max} \left\{ y = f(x) \right\} \tag{2.21}$$

where $f(\mathbf{x})$ can be any type of nonlinear function subject to the requirement of being differentiable. The general formulation of the problem allows for three types of constraints,

1. Equality constraints

$$\sum_{n=1}^{N} a_{kn} X_n = r_k \qquad for\ k = 1,2,....,K_e \tag{2.22a}$$

2. Inequality constraints

$$\sum_{n=1}^{N} a_{kn} X_n \geq r_k \qquad for\ k = K_{e+1},...,K \tag{2.22b}$$

3. Bounded variables

$$x_n^L \leq x_n \leq x_n^U \qquad for\ some\ of\ the\ x_n \tag{2.22c}$$

where the total number of control variables is N , the number of equality constraints is K_e , and the total number of constraints is K.

Except for the objective function, the above is similar to any standard linear programming (LP) or quadratic programming (QP) problem. The proposed procedure for solving the problem is to approximate the original nonlinear objective function by a quadratic function using Taylor series expansion about an initial feasible solution $\mathbf{x}^o$. Thus, the general Taylor series expansion for a nonlinear multivariate function reduces to,

$$f(x) - f(x^{\circ}) + [\nabla f(x^{\circ})]^T \partial x + \frac{1}{2} (\partial x)^T H(x^{\circ}) \partial x + 0 (\partial x^3) \tag{2.23}$$

where terms up to second-order have been considered explicitly, and where $\nabla f(\mathbf{x}^o)$ and $H(\mathbf{x}^o)$ represent the gradient vector and Hessian matrix of $f(\mathbf{x})$ at $\mathbf{x} = \mathbf{x}^o$, respectively.

Description of the Method

The method of solution described herein is devoted to solving this particular nonlinear programming problem, where the objective function is nonlinear and the constraints are linear. It is based on approximating the nonlinear objective function by a quadratic form and the problem being solved as a quadratic programming problem. A succession of these approximations is performed until the solution of the quadratic programming problem reaches the optimal solution. The following description was excerpted from the publication by Díaz and Fontane (1989).

Starting with an initial feasible solution $\mathbf{x}^0$, to be provided, the algorithm carries out a Taylor series expansion on the nonlinear objective function around the given initial solution, retaining the first and second order terms to form a quadratic function. This quadratic function is in turn converted into the standard quadratic form required for the solution based on a quadratic programming algorithm. The routine to be used was structured to solve minimization problems only. For that reason, the description of the method of solution given below corresponds to a minimization problem. Nevertheless, the proposed maximization problem can be solved by minimizing the negative of the objective to be maximized.

The search procedure for the minimization problem is accomplished by changing the value of a decision variable at each step (or level), such that the value of the approximated objective function is decreased while not violating the upper and lower bounds of the variables and any of the constraints. Furthermore, the magnitude of the move for the selected decision variable in QP is also limited by the possibility of change in sign in the constrained derivatives. At each step, the variable to be changed is the one inducing the largest change in the objective function per unit of change of that variable. The quadratic approximation of the objective function permits larger valid changes in the decision variables at each iteration in comparison to the case of a simple linear approximation. This results in a faster convergence to the optimal solution and lesser risk of solution divergence. The optimal solution within the bounded decision variables is said to be reached when a set of local conditions (necessary and sufficient) known as the Karush-Kuhn-Tucker conditions are satisfied, namely the complementary slackness.

However, the above optimal solution by standard QP is only true for the approximated objective function. Since the optimal values of the variables may differ from the initial values upon which the approximation of the nonlinear quadratic objective functions was based, it is necessary to repeat the described process by using the new values for the set of variables as the starting point for the next round of the sequential solution. This procedure, known as sequential quadratic

programming (SQP), is repeated until successive optimal values do not differ by more than the stipulated tolerance limit, or when the maximum limit on the number of sequences has been reached. Figure 2.6 illustrates the procedure outlined above but for the case of a maximization problem.

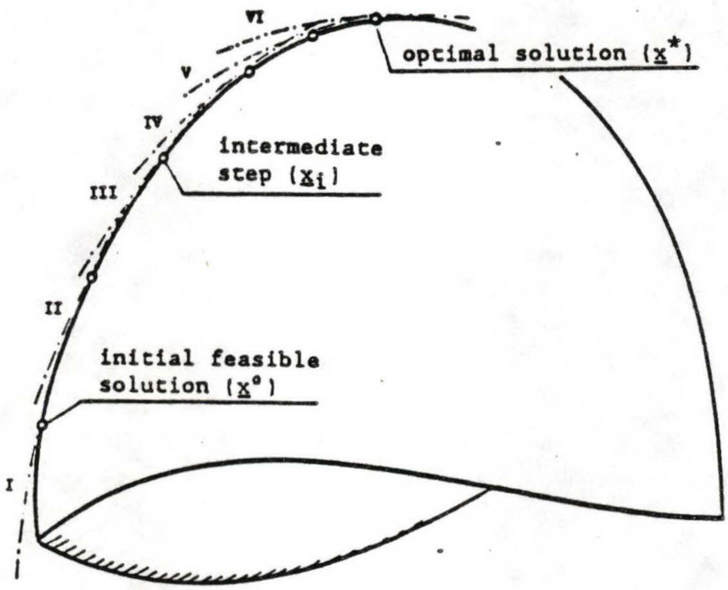

Fig 2.6 Maximization of a Concave Objective Function by SQP

A succession of these approximations is performed until the solution of the QP problem reaches the optimal solution. Due to the particular structure of the reservoir operation problem, a quadratic approximation of the objective function is advantageous, both from the theoretical and computational viewpoints. A quadratic approximation provides a close representation of the nonlinear objective function. Moreover, it permits larger valid changes in the control variables at each iteration in comparison to a linear approximation. This results in a faster convergence to the optimal solution and lower risk of solution divergence.

MODELING OF WATER-LOSSES

For above average hydrologic conditions, it may happen that a reservoir cannot temporarily store the difference between inflows and turbine (controlled) releases, due to high storage levels. In this situation, spillages (uncontrolled releases) have to take place to avoid dam overtopping. In a large system, the most upstream storage reservoirs attenuate inflow peaks and increase low flows to the maximum extent, absorbing the stochastic component of inputs. This effect permits the operation of the downstream portion of the system under more uniform conditions. Releases from upstream reservoirs, controlled and uncontrolled, have the opportunity to be restored for future use in the intermediate and downstream reservoirs of the system.

When formulating the optimization of the reservoir operation problem under the QP framework, the adoption of spillage releases as control variables in addition to turbine releases would double the dimension of the optimization problem. Besides the obvious complexity of handling a larger number of control variables, the computational burden is substantially increased. This more cumbersome formulation looks even less desirable if we consider that extremely wet periods of inflows may occur only for a few flow sequences when compared with the total amount of hydrologic traces being generated.

Several approaches have been developed to account for water losses. For example, in the work by Mariño and Loaiciga (1985), spillages are introduced implicitly in the formulation through the mass balance equation as a nonlinear function of storage, subject to the hydraulic properties of reservoir spills. This approach permits the reduction of the dimensionality of the decision space by linking spillages to storages, and solving for spillages and penstock releases simultaneously, at the expense of a laborious formulation.

In this study a different and simpler approach was adopted. It is based on considering evaporation losses and spillway releases as constant terms during the solution of the QP problem, and whose estimation is based on a previously obtained solution. The sequential nature of the optimization algorithm SQP, as explained in the previous subsection, permits the convergence of both terms toward their exact values at the same time that accounts for nonlinearities in the objective function. Also notice that by treating evaporation and spills as constants terms in the formulation, the nonlinear relationships between outflow-storage and evaporation-storage, typical of the reservoir dynamics, is in fact omitted, yielding a strictly linear set of constraints.

The reason for the success of this approach rests in the inherent nature of the objective function. The first derivative of the objective function with respect to turbine releases, a decreasing

function of marginal benefits versus increasing turbine releases, prevents uncontrolled releases from occurring as long as feasibility conditions permit. The objective function will force the allocation of excess inflow as turbine releases along the whole planning horizon rather than the concentration of large releases during few stages.

Chapter 3

SYNTHETIC GENERATION OF INFLOWS

IDENTIFICATION OF THE MODEL

The implicit stochastic optimization approach implemented as part of this study requires the synthetic generation of several series of flows at the monthly level. Stochastic generation models are widely utilized to synthetically generate unregulated inflows to a reservoir system, whose operation is then simulated or optimized. Various alternative models are suggested in the literature for modeling periodic multi-series, all of them encompassed in what is called the stochastic modeling of hydrologic series.

In general, hydrologic series have significant periodic behavior in the mean, standard deviation and skewness coefficients. In addition to these periodicities, they show a time correlation structure which may be either constant or periodic. The monthly optimization model requires a flow generative model which, besides preserving the mean and variance of the data series, must also preserve its correlation structure in time.

Another aspect to consider is the multivariate characteristic of the present problem. Since there are two locations for flow generation (basins A and B in Figure 1.3), the stochastic components of the two series may be dependent between themselves. When the objective is to generate new samples of time series at two locations, the basic requirement is not only to preserve the statistical characteristics at each station, but also to preserve the mutual space-correlation between the two time series. This requirement calls for a model considering both time and space characteristics at once. These types of models demand complex parameter estimation procedures, particularly for models of order higher than one.

In summary, time series modeling requires a systematic step-by-step approach to determine the most suitable stochastic model for the streamflow location under analysis. This procedure demands not only the proper identification and selection of the model type and form, but also includes testing procedures that guarantee the goodness of fit of the selected model, and the evaluation of parameters uncertainty (Salas et al., 1980). Obviously, as part of the testing procedure, the whole time series modeling process should be validated by the characteristics of the historic time series to be modeled.

However, as mentioned in Chapter 1, the entire model developed for this study is not intended for any particular geographic site. Then, it would be very unlikely to find a unique flow generation model that would be the most suitable one for all possible scenarios. For that reason, and considering the intended use of the generated data, it was preferred to avoid unnecessary complex flow generation models. In other words, there should be a trade-off between the complexity of the selected model and the expected accuracy of the generated flow series. Concluding, the time series model selected for this study is a univariate model of the autoregressive type with periodic parameters (PAR). The model has the capability to consider either lag-one (PAR-1) or lag-two (PAR-2) monthly serial correlations.

We may add that, as an alternative approach, the user may want to disregard the models included in this module and utilize any other available model of his preference. Externally from the optimization model, the user can generate all necessary traces of natural inflows and save them in a separate file, which then will be accessed sequentially by the optimization model to read the synthetic inflows.

DESCRIPTION OF THE FLOW GENERATION MODEL

This section presents the basic formulation for modeling periodic univariate time series based on AR models. Let's consider a periodic hydrologic series such as a monthly series $X_{v,t}$, where v denotes the year, t denotes the time interval within the year (monthly) with $t = 1,......,\omega$ and ω the number of time intervals within the year, $\omega = 12$. The skewed original series is approximately made symmetrical by means of a simple log-transformation. The new transformed series $Y_{v,t}$

$$Y_{v,t} = \log (X_{v,t}) \tag{3.1}$$

will prevent generating some negative flows during the synthetic generation process. The periodic (assumed normally distributed) series $Y_{v,t}$ is written as

$$Y_{v,t} = \mu_i + \sigma_t Z_{v,t} \tag{3.2}$$

where μ_t and σ_t are the periodic mean and periodic standard deviation, and $Z_{v,t}$ represents the time dependent series with mean zero and variance one. The time series $Z_{v,t}$ will be represented

by an AR model with periodic parameters, more specifically an AR(1) or AR(2) model, then

$$Z_{v,t} = \phi_{1,t} Z_{v,t-1} + \sigma_{\varepsilon,t} \xi_{v,t} \qquad \text{for AR(1)} \tag{3.3a}$$

$$Z_{v,t} = \phi_{1,t} Z_{v,t-1} + \phi_{2,t} Z_{v,t-2} + \sigma_{\varepsilon,t} \xi_{v,t} \qquad \text{for AR(2)} \tag{3.3b}$$

where the $\phi_{j,t}$'s are the periodic autoregression coefficients, $\sigma^2_{\varepsilon,t}$ is the periodic variance of the residuals, and $\xi_{v,t}$ is the independent standardized normal variable. The estimation of the periodic parameters reduces to:

1. the estimate of the periodic mean μ_t

$$\mu_t = Y_t = \frac{1}{N} \sum_{v=1}^{N} Y_{v,t} \qquad \text{for } t = 1, 2 \ldots\ldots, \omega \tag{3.4}$$

where N is the total number of years of available data

2. the estimate of the periodic standard deviation $\sigma_{\varepsilon,t}$

$$\sigma_t = s_t = \left[\frac{1}{N} \sum_{v=1}^{N} (Y_{v,t} - Y_t)^2 \right]^{1/2} \qquad \textit{for } t = 1, 2 \ldots\ldots, \omega \tag{3.5}$$

3. the estimate of the periodic autoregressive coefficients $\phi_{j,t}$. In particular for the AR(1) model

$$\phi_{1,t} = \rho_{1,t} \tag{3.6}$$

and for the AR(2) model

$$\phi_{1,t} = \frac{\rho_{1,t} - \rho_{1,t-1}\, \rho_{1,t}}{1 - \rho^2_{1,t-1}} \qquad \textit{for } t = 1, 2, \ldots\ldots, \omega \tag{3.7}$$

$$\phi_{2,1} = \frac{\rho_{2,t} - \rho_{1,t}\rho_{1,t-1}}{1 - \rho^2_{1,t-1}} \qquad \textit{for } t = 1, 2, \ldots\ldots, \omega \tag{3.8}$$

where $\rho_{k,t}$ is equal to the sample k^{th} serial correlation coefficient $r_{k,t}$ computed from the historical series.

Besides the series of preliminary tests to assess the goodness of fit of the selected model, like testing the independence of the residuals $\xi_{v,t}$, the model will be accepted as good if the model statistics resemble the historical ones. For instance, we may compare how well the historic periodic mean and standard deviation are reproduced by the synthetic generative model. Furthermore, the correlation structure is of capital importance if the synthetic series is to reproduce accurately the historic series. We may compare the general shape of $r_{k,t}$ with the shape of the mean of the correlograms $\rho_{k,t}$ generated from synthetic samples.

So far we have not addressed the fact that two flow series are being modeled simultaneously. As a simplification for the aforementioned problem, the adopted univariate model considers only two extreme cases regarding space-correlation, they are: (i) completely uncorrelated flow stations, and (ii) perfect space-correlation between the two stations. In the former case, two separate streams of independent stochastic components are maintained for each streamflow location, whereas for the latter case, the same $\xi_{v,t}$ series is used for generating monthly flows at both locations.

STREAMFLOW INCREASES

So far we have discussed the synthetic generation of the so called natural or normal flows, flows that are naturally originated in the basins. In contrast, artificial flows are those originated by some vegetation manipulation technique on some areas of the basin that creates streamflows in excess of what it would have existed otherwise.

It is not the purpose of this section to discuss the different existing techniques to generate artificial streamflow increases, nor to discuss the potential that a basin may have for augmenting water yield by manipulating vegetation. For those areas of interest, the reader is referred to the publications by Brown and Fogel (1987) and Brown et al. (1988). This section is only concerned with the quantification of runoff increases that have the capacity to reach storage reservoirs within the basin.

As reported by Brown et al. (1988), the annual fluctuation in flow increase is largely a function of the same hydrologic factors that affect normal flow. Different approaches have been adopted to estimate the runoff increases to be added to normal flows. The approaches mainly differ in how the per-acre estimate of annual runoff increase is obtained. Computation of the per-acre estimates are usually obtained based on:

(1) the "Proportional Approach", where annual flow increases are directly proportional to the annual flow,

(2) the "Logarithm Approach", where the natural logarithm of the return period of the annual flow is used as the basis of proportionality between the normal flow and the flow increase, and

(3) the "Precipitation Approach", when it is found that the flow response to vegetation management is more sensitive to variations in precipitation than variations in the normal flow.

The three methods require different levels of watershed data. For example, the precipitation approach requires a detailed knowledge of the relationship between normal runoff and precipitation, information not readily available. On the other hand, the proportional approach, only requires very basic information like historic mean annual flows at the site. As expected, the relationship between normal flow and increase flow is a point of considerable uncertainty. Therefore, not having a definite argument in favor of any one of the approaches, this study has adopted the simplest methodology, the proportional approach.

In general, the estimation of runoff increases follow the following steps: (1) estimate the annual runoff increase per-acre associated with the annual estimate of normal flow, (2) apportion the annual increase by month, and (3) extend the per-acre estimates to a watershed total based on estimates of treatable acreage within the study area. The three step procedure for computing monthly flow-increase volume can be summarized by the following equation,

$$I_{i,j} = I\,(Q_j/Q)\,P_i\,A \qquad (3.9)$$

where $I_{i,j}$ denotes the runoff-increase in volume units during month i of year j, **I** is the long term mean annual flow-increase per-acre, Q_j the predicted normal annual flow for the year j, **Q** the mean annual flow at the station, P_i the proportion of the annual flow-increase occurring during month i, and A represents the number of acres treated. For the model to compute the $I_{i,j}$ values, the user should provide the values of **I**, P_i, and A for all basins where vegetation management takes place. This information is included in file HYDRO.DAT as explained in the next section.

GENERAL DESCRIPTION OF MODULE-1

The program Module-1 constitutes the portion of the general model that computes the synthetically generated inflows to reservoirs A and C. The tree-diagram depicted by Figure 3.1 shows the subprograms contained within Module-1, and the sequence of calls among them. Three

main tasks are carried out by Module-1, they are:

- read time-series data pertinent to the hypothetical drainage basins, subroutine READF1,
- synthetically generate natural monthly flows at basins A and B, subroutine GENEAR,
- compute artificial flow-increases originated at sub-basins A and B, subroutine FLOINC.

```
|
|
|-------- READF1
|-------- GENEAR
|            |
|            |-------- STATIS
|            |-------- RANNOR
|                         |
|                         |-------- UNIF
|
|-------- FLOINC
|
|
```

Fig 3.1 Tree-Diagram of Module-1

A copy of the file read by subroutine READF1, named HYDRO.DAT, is included as Appendix I and described herein. The first level of data, indicated by the symbol "1-->" in the file, contains four variables that provide information about:

TRANSF	should a log-transformation of the historic flows be performed ?
INDEP	should the flow records at locations A and B be considered completely space-independent or vice-versa ?
AUTREG	should a PAR(1) or PAR(2) model be implemented ?
INCFLO	should natural inflows be augmented by artificial flow increases ?

Variables at the second level, AREA and AREB, should be considered as relative area factors that can proportionally modify the generated flows. For example, a value of 1.0 represents the actual size of the basin. A value 1.3 would indicate a basin 30% larger than what actually is, and consequently, would generate a 30% more flow than in the previous case.

At the third level, UNSYS allows the user to enter the historic flow records in english or metric units, and at the same time to indicate whether they represent flow or volume units. All choices are presented in the table to the right of UNSYS. At the fourth level, HMAFA and HMAFB represent the historic mean annual flow at branches A and B respectively, in the units

indicated by UNSYS. At the fifth and sixth levels, HMFA and HMFB provide the historic mean monthly flows at basins A and B respectively, also in the units indicated by UNSYS. Notice there are 12 columns for 12 monthly values and as many rows as years of historic record, indicated internally in the computer program by the Parameter statements NRA and NRB. The starting month can be any month provided that all the information is given consistently.

Starting at level 7, data concerning the generation of artificial flow-increases are provided. EFIA and EFIB are the long term expected annual flow-increase at both basins respectively. Both values are given as flow-depth (in inches) as customarily indicated in the literature. At level 8, ACTA and ACTB represent the total number of acres treated by any type of vegetation management technique resulting in runoff-increases at the upper portions of basins A and B respectively. Finally, at level 9, PIFA and PIFB provide the monthly percentages for distributing the corresponding total annual flow-increases along the year at both basins.

Once the reading of HYDRO.DAT is completed, subroutine GENERA is called upon to generate the unregulated portion of the total flows entering reservoirs A and C. Subroutine GENEAR very much follows the computational steps indicated earlier in the description of the generation model. According to the value assigned to variable AUTREG by the user, either a PAR model of order (1) or (2) will be utilized.

Two subroutines, STATIS and RANNOR, are in turn called by subroutine GENEAR. STATIS computes all the basic periodic statistics of the historical series required by GENEAR, namely: monthly mean, monthly standard deviation, and first and second serial correlation coefficients. Subroutine RANNOR generates the Normally distributed (0,1) random numbers used by GENEAR as the independent stochastic component of the model. Random numbers are generated by using the Box-Muller's formulae, which in turn calls for the computation of sequences of random numbers uniformly distributed over the unit interval, function UNIF. Before using the random number generator algorithm, the generator should be initialized by a seed or starting number using the computer system clock. The seed is requested only once during the execution of the program.

Finally, after the sequence of monthly flows is generated, natural flows can be augmented by the artificial flow-increases computed by subroutine FLOINC, if so required, depending on the value assigned to the variable INCFLO. In essence, the normal annual flow for a given year, as generated by subroutine GENERA, is used to compute the proportionality factor indicated in Eq.(3.9).

TESTING OF MODULE-1

In order to test MODULE-1 before being incorporated into the general model, it was utilized to generate monthly synthetic flows at two streamflow locations. The Collon River (basin A) and the Limay River (basin B) in the south-west region of Argentina were selected for that purpose. The way the two rivers are connected and the general conditions of the basin resemble the configuration of the hypothetical system discussed in Chapter 1. Thirty nine years of simultaneous record at both locations made possible the computation of the monthly statistics used by subroutine GENEAR to synthetically generate monthly flows. During the selected period of record both rivers carried unregulated flows with practically no disturbance of any kind.

Twenty five series of monthly flows for 39 years long each were generated using the PAR(1) and PAR(2) models. The average value of the statistics for each month for the 25 series were computed and compared with the parameters of the historical series. Tables 3.1 and 3.2, for the Collon and Limay Rivers respectively, summarize the results obtained. Furthermore, to ease the comparison, the same data is graphically displayed in Figures 3.2 and 3.3. The closeness of the results indicated that, at least for the chosen river system, the PAR model was an appropriate selection. Both models, PAR(1) and PAR(2), returned very similar results for this case, indicating the relatively short-term hydrological persistence of the Collon and Limay rivers.

It is known that when generating flows with a PAR model at a monthly level, there is no assurance that the statistical properties will also be preserved at the annual level. Fortunately for this case-study, mean and variance of mean annual flows were found very well preserved, although a more intensive testing procedure would be desirable.

Table 3.1 Collon River - Summary of Main Statistics

	1	2	3	4	5	6	7	8	9	10	11	12
Statistics from 39 years of historical record												
μ	108.	315.	568.	607.	627.	567.	652.	688.	484.	246.	139.	103.
σ	63.	241.	392.	292.	277.	180.	169.	196.	206.	109.	67.	64.
ρ	0.489	0.575	0.784	0.503	0.345	0.512	0.562	0.733	0.752	0.514	0.735	0.561
Statistics from 1000 years generated with PAR(1)												
μ	105.	300.	561.	601.	618.	559.	646.	680.	476.	245.	138.	102.
σ	77.	227.	439.	296.	284.	185.	161.	226.	247.	119.	70.	56.
ρ	0.497	0.596	0.551	0.652	0.462	0.563	0.566	0.734	0.815	0.691	0.816	0.753
Statistics from 1000 years generated with PAR(2)												
μ	110.	321.	581.	609.	632.	564.	649.	690.	488.	246.	138.	102.
σ	69.	287.	441.	282.	288.	183.	171.	238.	260.	119.	66.	55.
ρ	0.527	0.542	0.627	0.626	0.451	0.536	0.561	0.743	0.804	0.683	0.820	0.728

Monthly Period

where μ is the monthly mean
σ is the monthly standard deviation
ρ is the lag-1 serial correlation

Table 3.2 Limay River - Summary of Main Statistics

Monthly Period

	1	2	3	4	5	6	7	8	9	10	11	12
Statistics from 39 years of historical record												
μ	116.	192.	309.	358.	388.	363.	355.	376.	349.	274.	196.	143.
σ	48.	93.	150.	135.	125.	90.	70.	83.	107.	86.	67.	61.
ρ	0.790	0.735	0.734	0.710	0.605	0.740	0.753	0.887	0.822	0.758	0.909	0.793
Statistics from 1000 years generated with PAR(1)												
μ	115.	191.	305.	364.	394.	366.	355.	377.	351.	275.	195.	143.
σ	55.	99.	141.	144.	137.	98.	72.	91.	125.	104.	82.	71.
ρ	0.740	0.682	0.608	0.764	0.591	0.741	0.756	0.865	0.852	0.824	0.937	0.895
Statistics from 1000 years generated with PAR(2)												
μ	119.	194.	309.	352.	385.	362.	355.	376.	350.	275.	197.	143.
σ	58.	109.	162.	143.	132.	93.	69.	84.	115.	94.	75.	63.
ρ	0.724	0.653	0.659	0.737	0.584	0.727	0.743	0.852	0.838	0.801	0.918	0.873

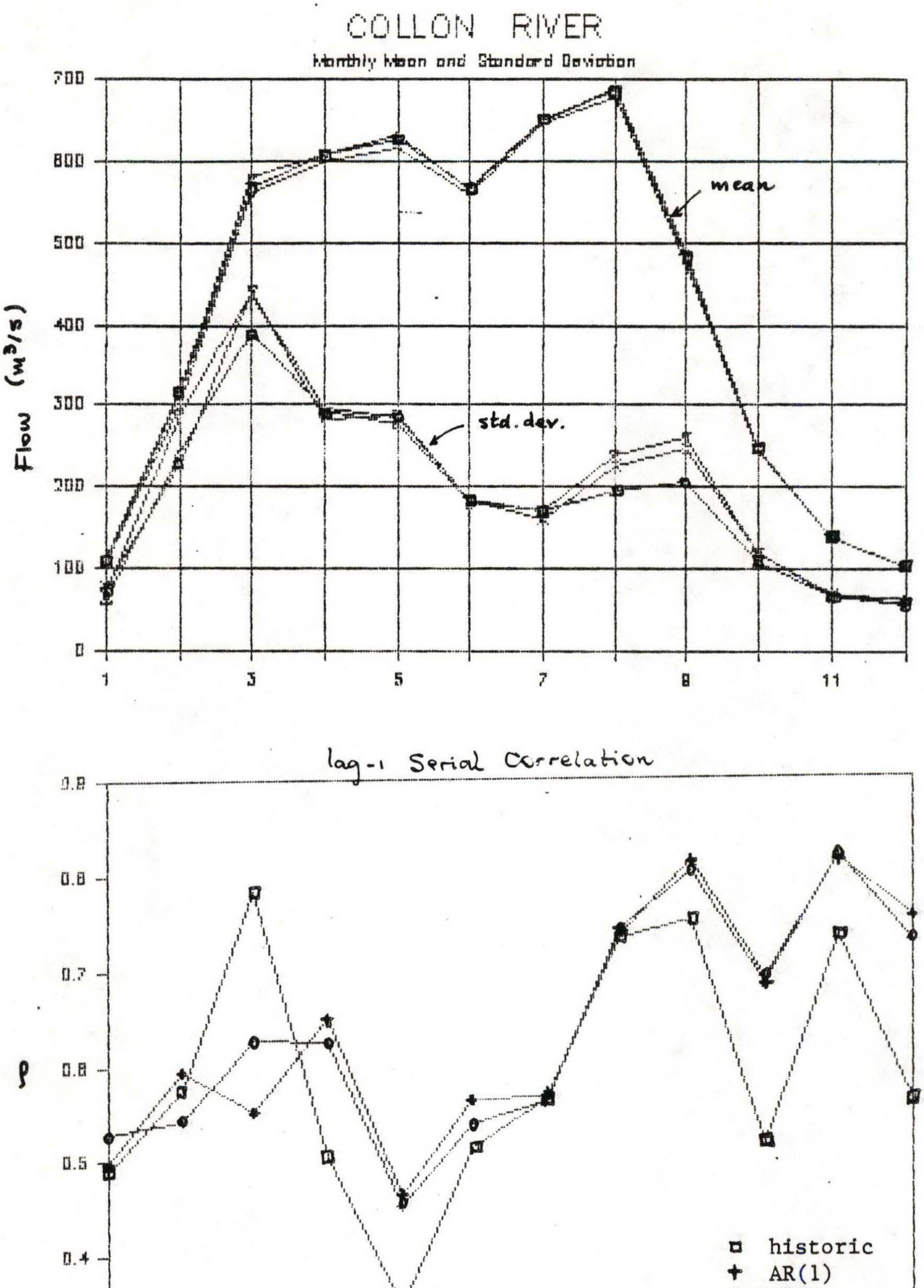

Fig 3.2 Collon River - Historic and Generated Statistics

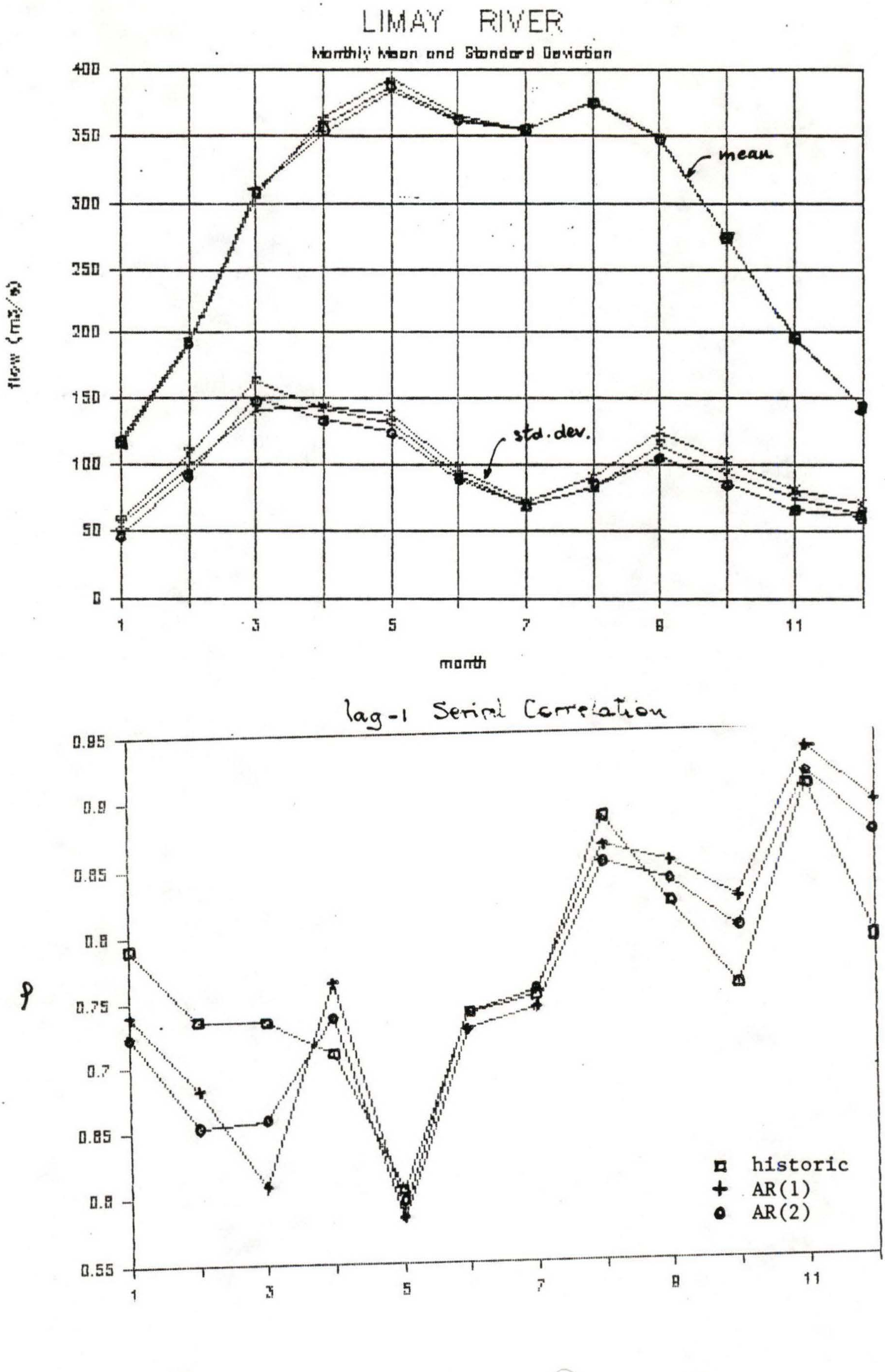

Fig 3.3 LIMAY River - Historic and Simulated Statistics

CHAPTER 4

SEARCH FOR AN INITIAL FEASIBLE SOLUTION

STATEMENT OF THE PROBLEM

The quadratic programming routine QPMOR, described in Chapter 5, based on the General Differential Algorithm, requires an initial feasible solution (IFS) to start the optimization process. This requirement has more importance than is usually attributed to, since IFS's are many times not easy to find for large and highly interacting systems. For this study, considering the large number of hydrologic traces to be analyzed, it becomes a necessity for the model to find the IFS's that feed the optimization algorithm. In other words, it would be unreasonable to have to provide (externally) the initial solution vector before the operation of the system is optimized for each hydrologic trace.

A possible approach to tackle this problem would be to run a simplified simulation model, previous to the optimization routine, to find any feasible state of the system. While this approach can be straight forward when working with average hydrologic conditions, the contrary is true when dealing with extreme conditions, particularly during very wet periods of inflow. Furthermore, the requisite that all reservoirs of the system should reach a certain storage level at the end of the operational horizon imposes a practically unsolvable problem for the simulation procedure.

For these reasons, this study has adopted a different approach. It is based on the concept behind the Two-Phase Method of Linear Programming, in which the so called Phase-1 is implemented first in order to obtain the required basic IFS for the original optimization problem. In this case, Phase-1 is used to find the required IFS for the operational problem, before actually solving the optimization problem via the quadratic programming algorithm. Although this approach is relatively more involved than a simple simulation procedure, the efficiency of the LP algorithm and the numerical stability of the Two-Phase Method assure a much better chance of finding an IFS under all flow conditions.

FORMULATION OF THE LP/PHASE-1 PROBLEM

The general differential algorithm, similar to the simplex method, starts with an initial feasible solution and moves to a series of improved solutions until the optimal point is reached, or unboundedness of the objective function is verified. In fact, the simplex method has the additional

requirement that all feasible solutions should also be basic. This section very succinctly describes a well known procedure, the Two-Phase Method, used to obtain the initial basic feasible solution that gets the QP optimization algorithm started.

In general, any linear programming formulation can be transformed into a problem of the following form

Minimize	$\mathbf{c\,x}$	(4.1a)
Subject to	$\mathbf{A\,x = b}$	(4.1b)
	$\mathbf{x \geq 0}$	(4.1c)

where $\mathbf{A}$ is the M×N coefficient matrix and $\mathbf{b \geq 0}$ is the nonnegative right-hand-side vector. This can be accomplished by introducing slack variables and by simple manipulations of the constraints and variables. Furthermore, if it can be ensured that $\mathbf{A}$ contains an identity matrix, then an immediate basic feasible solution is at hand. Otherwise, it is necessary to resort to artificial variables to obtain an initial basic feasible solution to a slightly modified set of constraints, and then use the simplex method itself to get rid of these artificial variables. The so-called Phase-1 of the Two-Phase method carries out this task.

During Phase-1, the LP restrictions set is changed by adding a vector of artificial variables $\mathbf{x_a}$ leading to the system $\mathbf{Ax + x_a = b}$, $\mathbf{x \geq 0}$, $\mathbf{x_a \geq 0}$. Note that by construction, the identity matrix corresponding to the artificial vector is being forced. This gives an immediate basic feasible solution of the new system, namely, $\mathbf{x_a = b}$, and $\mathbf{x = 0}$. However, even though an initial feasible solution now exists, the problem has in effect being changed. In order to get back to the original problem, the artificial variables should be forced to zero to attain feasibility in the original problem, because $\mathbf{Ax = b}$ if and only if $\mathbf{Ax + x_a = b}$ with $\mathbf{x_a = 0}$ (Bazaraa and Jarvis, 1977). The modified LP problem solved during Phase-1 with the basic feasible solution $\mathbf{x = 0}$ and $\mathbf{x_a = b}$ is of the form

Minimize	$\mathbf{1\,x_a}$	(4.2a)
Subject to	$\mathbf{A\,x + x_a = b}$	(4.2b)
	$\mathbf{x, x_a \geq 0}$	(4.2c)

The simplex method is used to minimize the sum of the artificial variables over the feasible region for the revised problem. The optimal solution for Phase-1 necessarily has all the artificial variables equal to zero, so the solution is feasible for the original problem as well. In other words, at the end of Phase-1 we either get a basic-feasible solution of the original problem, or $x_a \neq 0$, which implies that the original problem has no feasible solution.

Set of Constraint Equations

The operational constraints presented in Chapter 2 were formulated bearing in mind the physical understanding of how the system operates, but regardless of the computational algorithm to be working with. In this section, that set of constraint functions is specifically adapted to the requirements of the LP framework. The algorithm adopted for solving the LP/Phase-1 problem requires the constraint functions to be grouped in the following order: first, the less or equal group of inequalities [$\leq$]; second, the greater or equal group [$\geq$]; and third, the group of equality constraints [=]. Furthermore, all right-hand-sides (RHS) are to be positive.

The requirement of positive RHS's wouldn't represent a problem had not exist the possibility for the RHS's to change their signs depending on the inflow and outflow conditions. For instance, if a constraint function as initially formulated is known to always have a negative RHS, the requirement above mentioned is readily satisfied by reversing the inequality type, and consequently, turning positive the negative RHS. But unfortunately, for most of the constraints, it is not possible to conclude with certainty that the sign of the RHS will only be of one type or the other. Thus, it becomes necessary to check the sign of the RHS's before assembling any constraint function within the LP-Tableau. If a negative RHS is detected, signs at both sides of the inequality have to be reversed (as well as the inequality type), and that particular constraint function is being reassigned to its corresponding new group.

What follows herein are the expressions for the constraint equations as finally included in the LP/Phase-1 formulation. These equations contain the control variables T and D on the left-hand sides, and all constant terms (and variables assumed constant) on the right-hand sides. The equations are written for any time period i, and with all symbols as defined in Chapter 2.

Capacity Constraints: the RHS's as derived directly from Eq.(2.15) will most likely turn negative after a few time periods, hence, by reversing the inequality type, most of the RHS's will end up positive,

$$\sum_{k=1}^{i} \left(-T_k^{*}+D_k^{*}+T_k\right) \geq S^{\circ}-S_M+\sum_{k=1}^{i} \left(Q_k+L_k^{*}\right)-\sum_{k=1}^{i} \left(L_k+E_k\right) \tag{4.3}$$

Release Constraints: the RHS's as derived from Eq.(2.16) have the tendency to become negative immediately (practically since the beginning of the time sequence), then, reversing the inequality is convenient,

$$\sum_{k=1}^{i} \left(-T_k^* + D_k^* + T_k\right) \leq S^\circ - S_m + \sum_{k=1}^{i} \left(Q_k + L_k^*\right) - \sum_{k=1}^{i} \left(L_k + E_k\right) \tag{4.4}$$

Final Storage Constraints: the RHS's derived by Eq.(2.17) practically will always be negative, hence, a change of signs at both sides of the equality yields,

$$\sum_{k=1}^{i} \left(-T_k^* + D_k^* + T_k\right) = S^\circ - S^f + \sum_{k=1}^{i} \left(Q_k + L_k^*\right) - \sum_{k=1}^{i} \left(L_k + E_k\right) \tag{4.5}$$

Powerplant Constraints: the maximum turbine output from Eq.(2.18) is always a positive number, then,

$$T_i \leq T_M \tag{4.6}$$

Water-Supply Constraints: during the LP/Phase-1 stage, the volume of water diverted to the demand zones during any period i will be specified as a fixed number. As a further simplification, it is assumed that no water is diverted to the demand zones, then, Eq.(2.19) becomes,

$$D_i = 0 \tag{4.7}$$

Recreation Flow Constraints: for most hydrologic traces, the RHS's as derived from Eq.(2.20) will remain a positive number, then,

$$T_i - D_i \geq Qrec_i - L_i \tag{4.8}$$

Another obstacle during the computation of the constraints set is the dependance of most of the RHS's on spillages and/or evaporation losses, which are unknown before-hand. In order to start the computation process, both variables are arbitrarily initialized. Monthly net-evaporation is made equal to the historical net-evaporation for each month, and spillages are initially made equal to zero. The implications of this initialization procedure in the LP solution is fully addressed in a later section of this chapter.

GENERAL DESCRIPTION OF MODULE-2

A specific group of subroutines, contained within the so-called Module-2, performs the search for the required IFS. Module-2 returns to the main program a set of control variables satisfying the constraint set, that is later on used by the sequential quadratic programming algorithm (Module-3) to carry out the optimization. As indicated by the tree-diagram in Figure 4.1, Module-2 is made up of a total of 14 different subprograms. Three main tasks are carried out by Module-2, they are:

☛ read an input-data file containing information related to the reservoir system,

☛ find an initial feasible solution of the operational problem, and

☛ check that the returned IFS is truly feasible, before moving to the next module.

```
─┐
 ├──── READF2
 ├──── INISOL
 │       ├──── LPLHS
 │       ├──── LPCUMU
 │       ├──── LPRHS
 │       ├──── TABLEU
 │       ├──── SIMPLX
 │       │       ├──── SIMP1
 │       │       ├──── SIMP2
 │       │       └──── SIMP3
 │       │
 │       ├──── RENAME
 │       ├──── STATE
 │       └──── LPSPILL
 │               ├──── LPCUMU
 │               └──── STATE
 │
 ├──── FEASIB
 │
```

Fig 4.1 Tree-Diagram of Module-2

A. <u>Input of Reservoir System Data</u>: The first subprogram within Module-2, subroutine READF2, reads in an input file which specifies the physical and operational characteristics of the reservoir

system. An example of the input file, named SYSTEM.DAT, is included as Appendix II and briefly described herein. The information provided by this file is organized in columns and rows. Data in the first column correspond to reservoir A while the central column applies to reservoir C. A third column provides the measuring units for each entry.

Levels 1 and 2 (1--> and 2--> as identified in file SYSTEM.DAT) provide the maximum and minimum operational storage allowed at the reservoirs, respectively. Levels 3 and 4 indicate the required reservoir storages at the beginning and end of the operational horizon, respectively. At level 5, the storage capacities at the spillway crest level are indicated. Levels 6 and 7 provide the coefficients of the exponential models representing: Reservoir-Storage vs Uncontrolled-Outflow (over the spillway); and Reservoir-Storage versus Water-Elevation, respectively. In fact, given the monthly time steps adopted in this study, unlimited outlet capacity is assumed when the reservoir reaches the value of storage that corresponds to the spillway crest level. Thus, unless the time step is drastically reduced, the information provided at levels 6 and 7 is ignored. The coefficients of the exponential model that relates Reservoir-Area vs Reservoir-Volume, used by the model to compute net-evaporation from the reservoirs, are provided at level 8. At level 9, the monthly rates of net-evaporation (evaporation minus rainfall) are provided. Levels 10 and 11 relate to the hydropower generating facilities. Level 10 indicates the maximum powerplant discharge (in volume units) operating continuously during the whole time interval, while level 11 indicates the overall efficiency of the generating facilities, excluding turbine efficiency. The last level, 12, specifies the minimum monthly flow required for recreational purposes, downstream from the diversion points.

B. Main Subprogram: The next subprogram in line with READF2, subroutine INISOL, performs all the sequential calls leading to the attainment of the IFS. It searches for the required IFS by solving Phase-1 of an LP problem, subject to a sub-set of the general constraint set governing the operation of the hypothetical reservoir system.

C. Computation of Constraint Coefficients: The first task for subroutine INISOL is to prepare a two-dimensional array, which translates into the so called LP-Tableau format, the set of constraint functions. The computation of the coefficients of the LP constraint set is carried out by two different subroutines, LPLHS and LPRHS, listed in Figure 4.1. LPLHS computes the left-hand-side matrix coefficients of the linear constraints, whereas LPRHS computes the right-hand-side terms. The reason for splitting the total task between the two subroutines is because LPLHS has to be called only once during the whole execution of the model, since the matrix coefficients remain the same for all hydrologic traces. On the contrary, LPRHS needs to be called at least once for every

new hydrologic trace, since most of the right-hand-side terms are function of the incoming flows. In between LPLHS and LPRHS, Figure 4.1 includes subroutine LPCUMU. This is just an auxiliary subroutine, written with the purpose of expediting the computations in most of the subprograms called by INISOL.

D. Assembling the LP Tableau: Once all terms of the constraint functions are computed, subroutine TABLEU is called for assembling left- and right-hand-side coefficients into the LP-Tableau format. It is under subroutine TABLEU that the program checks for the positiveness of all RHS's, and assigns the constraint functions to one of the three possible groups according to their form, the [≤], the [≥], or the [=] groups. The entries inside the tableau are nothing else than the coefficients of the original problem, as posed by Eqs (4.3) to (4.8), organized in a tabular form. These entries, together with the number of control variables, number of constraints, and number of constraints within each group, constitute the inputs needed by subroutine SIMPLX to solve Phase-1 of the overall LP problem.

E. Solution of LP/Phase-1: Subroutine SIMPLX, based algorithmically on the implementation of the Simplex Method by Kvenzi, Teschach, and Zehnder (Press, 1986, Section 10.8), is a very efficient routine requiring relatively little memory storage to operate. In turn, subroutine SIMPLX calls subroutines SIMP1, SIMP2, and SIMP3 to carry out all the specific matrix manipulations embedded in the Simplex method. As output, subroutine SIMPLX returns to INISOL the indication that a finite solution has been found, or not, and the two-dimensional tableau from where the solution vector is extracted, if applicable.

F. Computing the State of the System: Immediately after a feasible solution from Phase-1 is obtained, the program calls subroutine RENAME, which desegregates the string of control variables (the solution vector) into its four more physically based components, they are: turbine releases from powerplants A and C and water diversions to demand zones A and C respectively. Even though this step is not strictly necessary, it facilitates the programming and makes the code more readable for any future modification. Next subprogram being called is subroutine STATE, which computes the reservoir storage trajectories (state of the system) based on the monthly series of inflows and outflows to the reservoirs, as derived from the obtained LP/Phase-1 solution.

G. Detecting Reservoir Spills: It should be remembered at this point that the LP problem, as formulated, does not account for reservoir spills explicitly. While this approach greatly simplifies the LP formulation, it requires the implementation of an iterative procedure that permits the model to differentiate between powerplant releases and the unavoidable spillages that will take place

during periods of very large inflows. A detailed discussion of this iterative-procedure is presented in the next section. Nevertheless, we can anticipate here that subroutine LPSPILL, called by INISOL after computing the state of the system, is in charge of detecting and computing the reservoir spills that could eventually be masked in the solution vector provided by LP/Phase-1.

H. Checking Feasibility of the IFS: Finally, the last subprogram being called within Module-2, subroutine FEASIB, double checks the feasibility of the attained initial solution. This apparent redundancy in checking the feasibility of a solution that came out feasible from LP/Phase-1, is recommended because of the aforementioned iterative procedure instrumented within INISOL for detecting spillages.

ITERATIVE PROCEDURE TO FIND THE IFS

This section describes in detail the sequence of steps implemented by subroutine INISOL to reach an initial solution that is not only feasible, but also, as close as possible to the final (optimized) state of the system. This requirement generally translates into IFS's where reservoir spillages are avoided as much as possible (water losses), and regulated releases are approximately evenly distributed during the planning horizon due to the characteristics of the hydro-energy market. However, since there is no objective function driving the LP/Phase-1 solution to any particular state of the system (remember that LP/Phase-1 is solved using an auxiliary objective function, Eq.(4.2a)), it becomes necessary to force any desirable state of the system by only working with the constraints set.

As a first attempt, subroutine INISOL searches for an IFS free of spillages. In order to do that, it restricts, by means of the constraint set, the feasible reservoir operation band in between the top of the inactive pool (considered as minimum storage), and the top of the conservation pool at the spillway crest level (considered as maximum storage). By not allowing the water level in the reservoir to rise above the spillway crest level, reservoir spills are indirectly eliminated from the IFS. If after solving LP/Phase-1 under these restricted conditions a feasible solution is found, then the solution vector is extracted from the LP-tableau returned by subroutine SIMPLX, and the main task of subroutine INISOL is practically over.

However, when the inflow series are characterized by periods with very wet years, the restriction in the maximum storage capacity mentioned above may cause no solution to satisfy the given set of constraints. For this last case, it is necessary to reformulate the LP problem in more realistic terms. This reformulation translates in that an IFS can be obtained at the expense of

allowing spillages from the reservoirs. Computationally speaking, the maximum storage capacities at the reservoirs have to be reset to their original values (above the spillway crest), the right-hand-side terms have to be recomputed, and Phase-1 of the LP problem has to be run again for the second attempt to find an IFS.

If an IFS was found during the first attempt of LP/Phase-1, there should be no inconsistency between the resulting state of the system and the imposed feasible conditions upon which the solution was found, and as expressed earlier, the IFS can then be turned to the SQP algorithm. However, if a second run of LP/Phase-1 is necessary to obtain the IFS, the state of the system as computed by subroutine STATE will reveal that water is held in storage even above the uncontrolled spillway crest level (physically impossible), as illustrated in Figure 4.2 with the dotted line. This situation occurs because, during the second LP attempt, it is assumed that the conservation storage extends above the spillway crest level, in other words, it would be like considering positive semi-infinite reservoirs.

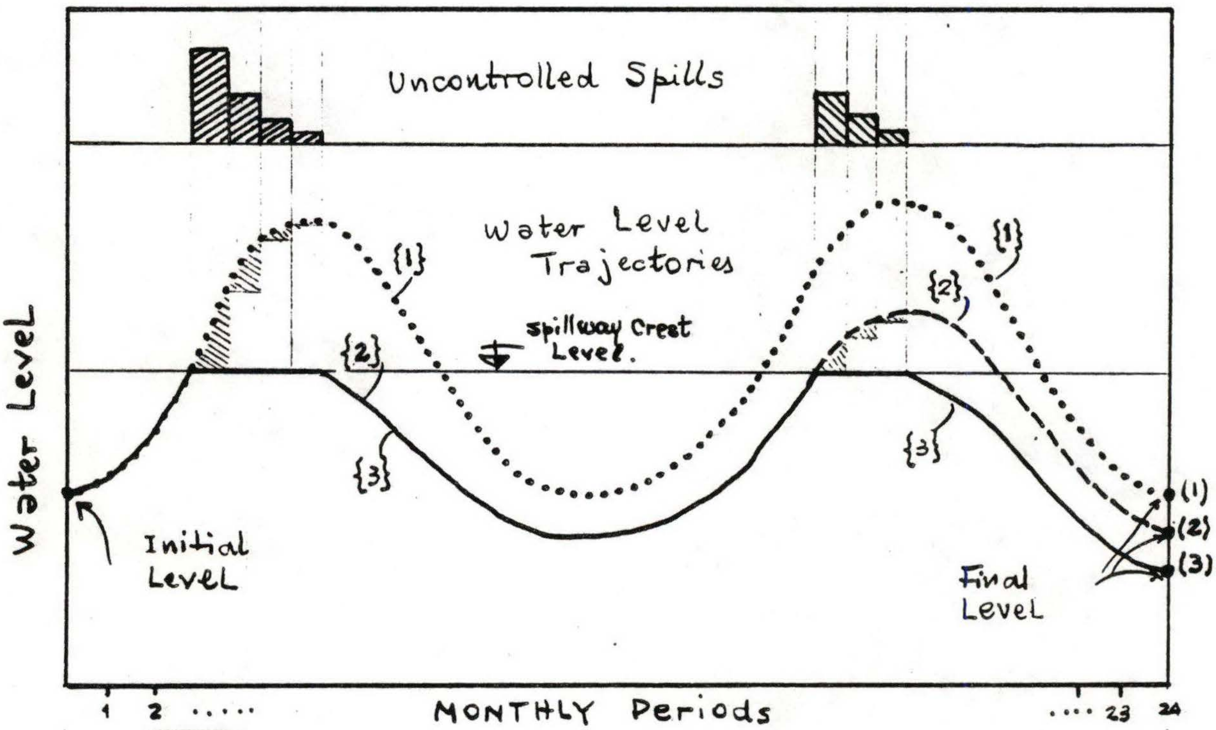

Fig 4.2 State of the System Before and After LPSPILL

Nevertheless, by running subroutine LPSPILL, the volume of water held artificially in storage (over the spillway) is transformed into spills from the reservoir. Consecutive computations of spillages and flow-routing through the reservoir have to be performed by subroutine LPSPILL to properly compute spills, in such a way that no water is kept in storage above the spillway crest level. For instance, the hypothetical dotted trajectory {1} in Figure 4.2 depicts two peaks of storage in excess of spillway crest storage. Starting with the first crest, LPSPILL computes the corresponding amount of spill. Immediately after the computation of the first sequence of spills is completed, the state of the system is recomputed, giving place to the trajectory individualized by the dashed line {2}. Now, even though the first crest has been eliminated, part of the second crest still remains. The procedure is repeated one more time to account for the second crest of over-storage, finally yielding the trajectory depicted by the solid line {3}. During any time period i, the amount of spill L_i is computed by:

$$L_i = \sum_{k=1}^{i} (INF_k - T_k - E_k) - \sum_{k=1}^{i-1} L_k \tag{4.9a}$$

provided that: 1) the water level in the reservoir is above the spillway crest level; 2) total inflows exceeds total outflows; and 3) the turbines release has reached its maximum capacity during the period in question; otherwise:

$$L_i = 0 \tag{4.9b}$$

By now, the IFS as modified by subroutine LPSPILL (trajectory {3} in Figure 4.2) does not fully satisfy the original constraint set. There are two departures from the original conditions. First, the pre-assumption of zero spills as initially assumed is no longer true, and second, the prescribed storage at the end of the planning horizon is not satisfied. Looking again at Figure 4.2, it can be seen that the final storage level has dropped from its original location at level (1), to (2) and (3) consecutively. Obviously, the difference in storage between levels (1) and (3) should account for the sum of all spills computed by subroutine LPSPILL. In order to regain feasibility, the LP/Phase-1 problem has to be solved one more time. For this third attempt, the computation of the RHS's should incorporate the spillages computed by LPSPILL as indicated in the previous paragraph. In this manner, spills are accounted as a water loss before re-solving the LP problem. Having indirectly eliminated that extra volume of water from the reservoir, there should be no inconvenience for the LP algorithm to attain an IFS that fully satisfies the feasible conditions, and

moreover, the requirement that the solution should be contained within the conservation pool.

Summarizing, either **one** or **three** LP/Phase-1 attempts may be necessary to find a particular IFS. For the majority of the inflow traces (those not very different from average hydrological conditions), solving the LP/Phase-1 problem only once will be enough to find the required IFS. For less typical inflow series (very wet years), all three steps will be necessary. The first attempt will be infeasible, the second attempt will permit the model to detect all the excess water that prevented the algorithm from reaching an IFS during the first attempt, and finally the third attempt, where a truly feasible solution will be obtained.

TESTING OF MODULE-2

This section presents a numerical example that illustrates all the steps implemented by the program Module-2. In order to fully test the capability of the module to find IFS's under any inflow condition, an extreme case was analyzed. Extreme conditions can be simulated by the model in two different ways; by arbitrarily increasing the inflows to the system, or by arbitrarily reducing the storage and outflow capacity of the reservoir system. In fact, a combination of the two was used for testing the module. As a first imposition, the maximum powerplant capacity at all months was made equal to the monthly average of total inflows minus evaporation. In this manner, since initial and final storage have to be equal, turbine discharge is forced to reach maximum capacity during all periods. Second, by reducing the storage volume corresponding to the spillway crest level (level 5 in file SYSTEM.DAT), the water level is forced to rise above the spillway crest, generating reservoir spills.

The input file SYSTEM.DAT presented in Appendix II was used for running the test problem. Notice in that file that the maximum turbine output was set equal to 1232 and 1950 MCM/month at powerplants A and C respectively. Initial and final storage are the same and equal to 4000 and 9000 MCM respectively at reservoirs A and C, and storage volumes at the spillway crest level are 5500 and 9500 MCM respectively. The sketches in Figure 4.3 summarizes the main operational levels at the two reservoirs. As expected, no feasible solution was found during the first attempt in solving the LP problem. Even with the powerplants operating continuously at maximum capacity, the storage trajectories can not be contained within the conservation zones. Table 4.1 shows the solution returned by the second LP/Phase-1 attempt, where i denotes the monthly period, Q unregulated inflow, E net-evaporation, T turbine release, L reservoir spillages, D off-stream water diversion, and S end-of-period reservoir storage. All magnitudes represent

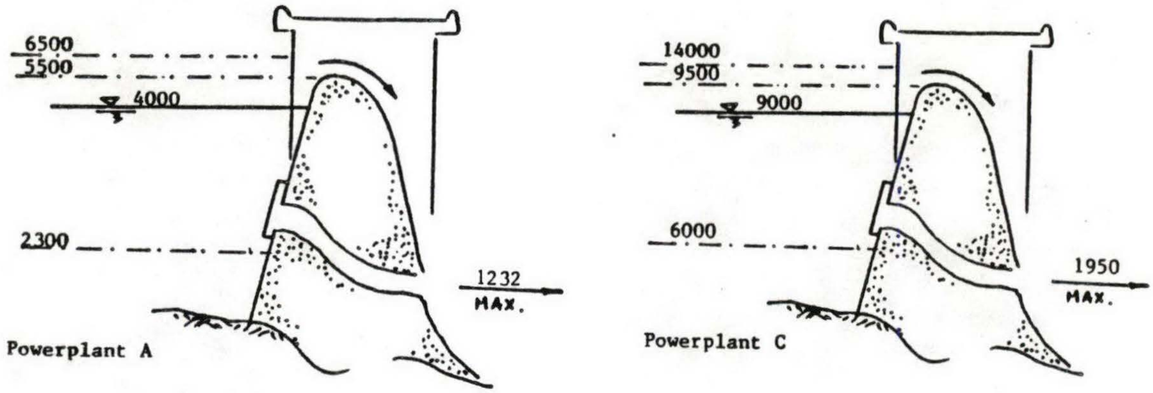

Fig 4.3 Operational Levels at Reservoirs A and C for LP/Phase-1

Table 4.1 State of the System After Second LP/Phase-1 Attempt

	Reservoir A						Reservoir C					
i	Q	E	T	L	D	S	Q	E	T	L	D	S
0						4000.						9000.
1	900.	29.	1232.	0.	0.	3639.	400.	56.	1950.	0.	0.	8626.
2	1100.	25.	1232.	0.	0.	3482.	500.	46.	1950.	0.	0.	8362.
3	1600.	23.	1232.	0.	0.	3827.	800.	39.	1950.	0.	0.	8405.
4	1800.	12.	1232.	0.	0.	4383.	1000.	23.	1950.	0.	0.	8664.
5	1600.	12.	1232.	0.	0.	4739.	1000.	16.	1950.	0.	0.	8930.
6	1400.	12.	1232.	0.	0.	4895.	900.	16.	1950.	0.	0.	9096.
7	1700.	11.	1232.	0.	0.	5352.	900.	14.	1950.	0.	0.	9264.
8	1800.	11.	1232.	0.	0.	5909.	1000.	16.	1950.	0.	0.	9530.
9	1400.	12.	1232.	0.	0.	6065.	900.	21.	1950.	0.	0.	9691.
10	800.	20.	1232.	0.	0.	5613.	700.	35.	1950.	0.	0.	9638.
11	500.	21.	1232.	0.	0.	4860.	500.	44.	1950.	0.	0.	9376.
12	400.	30.	1232.	0.	0.	3998.	400.	56.	1950.	0.	0.	9002.
13	900.	29.	1232.	0.	0.	3637.	400.	56.	1950.	0.	0.	8628.
14	1100.	25.	1232.	0.	0.	3480.	500.	46.	1950.	0.	0.	8364.
15	1600.	23.	1232.	0.	0.	3825.	800.	39.	1950.	0.	0.	8407.
16	1700.	12.	1232.	0.	0.	4281.	900.	23.	1950.	0.	0.	8566.
17	1600.	12.	1232.	0.	0.	4637.	1000.	16.	1950.	0.	0.	8832.
18	1800.	12.	1232.	0.	0.	5193.	1200.	16.	1950.	0.	0.	9298.
19	2200.	11.	1232.	0.	0.	6150.	1200.	14.	1950.	0.	0.	9766.
20	2100.	11.	1232.	0.	0.	7007.	1000.	16.	1950.	0.	0.	10032.
21	1400.	12.	1232.	0.	0.	7163.	800.	21.	1950.	0.	0.	10093.
22	300.	20.	1232.	0.	0.	6211.	700.	35.	1950.	0.	0.	10040.
23	200.	21.	1232.	0.	0.	5158.	300.	44.	1950.	0.	0.	9578.
24	100.	30.	1228.	0.	0.	4000.	200.	56.	1950.	0.	0.	9000.

volume in millions of cubic meters (MCM). Notice from Table 4.1 that turbine releases are at their maximum as predicted, all spills values are zero (as they are forced to be during the second LP attempt), and that reservoir storages at some periods are above the spillway crest level (highlighted values). Furthermore, initial and final storages are equal to the prescribed values.

All the storage volume exceeding the top of the conservation pool capacity has to be converted into uncontrolled spills. As mentioned in the previous section, the detection and computation of spills masked in the LP solution are carried out by subroutine LPSPILL, immediately after the second LP/Phase-1 attempt. Table 4.2 shows the state of the system after the program finishes running LPSPILL. Notice now from Table 4.2 that there isn't any end of period storage with a value higher than 5500 or 9500 MCM at reservoirs A and C respectively. The difference in volume between the two storage trajectories given in Tables 4.1 and 4.2 is released from the reservoirs in the form of uncontrolled spills. Also notice that the storages at the end of the 24th month, 2337 and 8407 MCM respectively, do not satisfy the requirement of being equal to the initial storages, 4000 and 9000 MCM.

Table 4.2 State of the System After Subroutine LPSPILL

	Reservoir A						Reservoir C					
i	Q	E	T	L	D	S	Q	E	T	L	D	S
						4000.						**9000.**
1	900.	29.	1232.	0.	0.	3639.	400.	56.	1950.	0.	0.	8626.
2	1100.	25.	1232.	0.	0.	3482.	500.	46.	1950.	0.	0.	8362.
3	1600.	23.	1232.	0.	0.	3827.	800.	39.	1950.	0.	0.	8405.
4	1800.	12.	1232.	0.	0.	4383.	1000.	23.	1950.	0.	0.	8664.
5	1600.	12.	1232.	0.	0.	4739.	1000.	16.	1950.	0.	0.	8930.
6	1400.	12.	1232.	0.	0.	4895.	900.	16.	1950.	0.	0.	9096.
7	1700.	11.	1232.	0.	0.	5352.	900.	14.	1950.	0.	0.	9264.
8	1800.	11.	1232.	**409.**	0.	5500.	1000.	16.	1950.	**439.**	0.	9500.
9	1400.	12.	1232.	**156.**	0.	5500.	900.	21.	1950.	**317.**	0.	9500.
10	800.	20.	1232.	0.	0.	5048.	700.	35.	1950.	0.	0.	9447.
11	500.	21.	1232.	0.	0.	4295.	500.	44.	1950.	0.	0.	9185.
12	400.	30.	1232.	0.	0.	3433.	400.	56.	1950.	0.	0.	8811.
13	900.	29.	1232.	0.	0.	3072.	400.	56.	1950.	0.	0.	8437.
14	1100.	25.	1232.	0.	0.	2915.	500.	46.	1950.	0.	0.	8173.
15	1600.	23.	1232.	0.	0.	3260.	800.	39.	1950.	0.	0.	8216.
16	1700.	12.	1232.	0.	0.	3716.	900.	23.	1950.	0.	0.	8375.
17	1600.	12.	1232.	0.	0.	4072.	1000.	16.	1950.	0.	0.	8641.
18	1800.	12.	1232.	0.	0.	4628.	1200.	16.	1950.	0.	0.	9107.
19	2200.	11.	1232.	**85.**	0.	5500.	1200.	14.	1950.	**160.**	0.	9500.
20	2100.	11.	1232.	**857.**	0.	5500.	1000.	16.	1950.	**1123.**	0.	9500.
21	1400.	12.	1232.	**156.**	0.	5500.	800.	21.	1950.	**217.**	0.	9500.
22	300.	20.	1232.	0.	0.	4548.	700.	35.	1950.	0.	0.	9447.
23	200.	21.	1232.	0.	0.	3495.	300.	44.	1950.	0.	0.	8985.
24	100.	30.	1228.	0.	0.	**2337.**	200.	56.	1950.	0.	0.	**8407.**

Complete feasibility is regained by running LP/Phase-1 for the third time. The spill values shown in Table 4.2 are explicitly accounted for during the computation of the RHS's for the new run. In this manner, the excess water is part now of the LP formulation as a water loss. Results from the third run are shown in Table 4.3. Notice that full feasibility is reached now that final storage equals initial storage. This condition marks the end of Module-2 since an initial feasible solution was obtained.

Table 4.3 State of the System After Third LP/Phase-1 Attempt

	Reservoir A						Reservoir C					
i	Q	E	T	L	D	S	Q	E	T	L	D	S
						4000.						**9000.**
1	900.	29.	1232.	0.	0.	3639.	400.	56.	1950.	0.	0.	8626.
2	1100.	25.	1232.	0.	0.	3482.	500.	46.	1950.	0.	0.	8362.
3	1600.	23.	1232.	0.	0.	3827.	800.	39.	1950.	0.	0.	8405.
4	1800.	12.	1232.	0.	0.	4383.	1000.	23.	1950.	0.	0.	8664.
5	1600.	12.	1232.	0.	0.	4739.	1000.	16.	1950.	0.	0.	8930.
6	1400.	12.	1232.	0.	0.	4895.	900.	16.	1950.	0.	0.	9096.
7	1700.	11.	1232.	0.	0.	5352.	900.	14.	1950.	0.	0.	9264.
8	1800.	11.	1232.	409.	0.	5500.	1000.	16.	1950.	439.	0.	9500.
9	1400.	12.	1232.	156.	0.	5500.	900.	21.	1950.	317.	0.	9500.
10	800.	20.	1232.	0.	0.	5048.	700.	35.	1950.	0.	0.	9447.
11	500.	21.	1232.	0.	0.	4295.	500.	44.	1950.	0.	0.	9185.
12	400.	30.	1232.	0.	0.	3433.	400.	56.	1950.	0.	0.	8811.
13	900.	29.	1232.	0.	0.	3072.	400.	56.	1950.	0.	0.	8437.
14	1100.	25.	1232.	0.	0.	2915.	500.	46.	1950.	0.	0.	8173.
15	1600.	23.	1232.	0.	0.	3260.	800.	39.	1950.	0.	0.	8216.
16	1700.	12.	1232.	0.	0.	3716.	900.	23.	1950.	0.	0.	8375.
17	1600.	12.	1232.	0.	0.	4072.	1000.	16.	1950.	0.	0.	8641.
18	1800.	12.	1232.	0.	0.	4628.	1200.	16.	1950.	0.	0.	9107.
19	2200.	11.	1232.	85.	0.	5500.	1200.	14.	1950.	160.	0.	9500.
20	2100.	11.	1232.	857.	0.	5500.	1000.	16.	1950.	1123.	0.	9500.
21	1400.	12.	1232.	156.	0.	5500.	800.	21.	1950.	217.	0.	9500.
22	300.	20.	1232.	0.	0.	4548.	700.	35.	1897.	0.	0.	9500.
23	200.	21.	597.	0.	0.	4130.	300.	44.	853.	0.	0.	9500.
24	100.	30.	200.	0.	0.	**4000.**	200.	56.	844.	0.	0.	**9000.**

Module-2 was tested in a personal computer with a 80386 Intel processor running at 25 Mhz. The module was compiled using the Microsoft-Fortran compiler Version 2.3. The total execution time for the three LP/Phase-1 successive runs was approximately 8 minutes.

CHAPTER 5

SEQUENTIAL QUADRATIC OPTIMIZATION

STATEMENT OF THE PROBLEM

The realistic conceptualization of the present operational problem demands the consideration of nonlinear terms in the objective function. Besides the nonlinearities introduced in the model when realistically modeling hydropower generation, many economists have found that also some degree of nonlinearity is more the rule rather than the exception in the economic considerations of planning and operational problems of this type. The return from water allocation when affected by the law of diminishing returns, as utilized in Chapter 2, represents a typical case.

First, this chapter presents the formulation of the nonlinear optimization problem within the framework of quadratic programming, including the set of constraints that guide the final solution. The following subsection analyzes in detail aspects of the indirect approach used by the model to account for water losses. Next, a general description of the Module-3 is presented, with step by step comments on all the subprograms implemented under this module. Finally, a test problem for Module-3 is presented. The results are discussed, as well as the input data used to run the test problem. A general indication of the computation time required by Module-3 is included at the end of the chapter.

FORMULATION OF THE QP PROBLEM

The standard form of a QP problem in matrix notation is defined by

$$Max\left[y(x) = w + c^T x + \frac{1}{2} x^T Q\, x\right] \tag{5.1a}$$

subject to the linear constraints,

$$g(x) = A\,x \geq r \tag{5.1b}$$

and nonnegativity conditions,

$$x \geq 0 \tag{5.1c}$$

where w is a scalar, the {c} and {r} vectors have known components, [A] is the matrix of constraint coefficients, and [Q] is a square symmetric matrix. On the other hand, the second order approximation of the original objective function given by Eq.(2.23) can also be expressed as

$$f(x) \approx f(x^o) + \sum_{i=1}^{N} D_i (x_i - x_i^o) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} H_{ij} (x_i - x_i^o)(x_j - x_j^o) \tag{5.2}$$

where D_i = the i^{th} element of the gradient vector $\partial f / \partial x_i$; H_{ij} = the element in the i^{th} row and jth column of the Hessian matrix $\partial^2 f / \partial x_i \partial x_j$ at $\mathbf{x}=\mathbf{x}^o$; x_i = the i^{th} element of the control vector $\mathbf{x}$; x_i^o = the i^{th} element of the initial solution $\mathbf{x}^o$; and N the total number of variables.

In order to conform to the standard quadratic form it is necessary to derive the {c} vector and [Q] matrix from the second order approximation given in Eq.(5.2). This is readily achieved by expanding Eq.(5.2) and interchanging the role of indices i and j in the linear terms. Ultimately, the following expression is obtained

$$f(x) \approx f(x^o) - \sum_{i=1}^{N} D_i x_i^o + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} H_{ij} x_i^o x_j^o + \sum_{i=1}^{N} x_i \left(D_i - \sum_{j=1}^{N} H_{ij} x_j^o \right)$$

$$+ \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} H_{ij} x_i x_j \tag{5.3}$$

The standard form of a QP problem as given by Eq.(5.1a) can also be written in the following expanded form

$$Max \left[y(x) = w + \sum_{i=1}^{N} c_i x_i + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i q_{ij} x_j \right] \tag{5.4}$$

By direct comparison of Eq.(5.3) with Eq.(5.4) it is possible to individualize the components of w, the {c} vector, and the [Q] matrix as follows:

$$w = f(x^o) - \sum_{i=1}^{N} D_i x_i^o + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} H_{ij} x^o x_j^o \tag{5.5a}$$

$$c_i = \left(D_i - \sum_{j=1}^{N} H_{ij} x_j^o \right) \qquad for\ i = 1,2,....,N \tag{5.5b}$$

$$q_{ij} = H_{ij} \qquad for\ i = 1,2....,N \tag{5.5c}$$

Once the equivalent {c} vector and [Q] matrix have been explicitly defined, a standard QP code can be used to solve for the set of x_i values which maximizes the objective function. Constant terms given by (5.5a) are omitted in the optimization.

DIFFERENTIATION OF THE OBJECTIVE FUNCTION

The expressions for the gradient vector $\nabla f(\mathbf{x}^o)$ and the Hessian matrix $\mathbf{H}(\mathbf{x}^o)$ require the computation of a series of first, second and cross partial derivatives of the objective function with respect to the control variables. For this type of problem, involving many variables and a complex objective function, a finite difference scheme appears like an expeditiousness procedure to compute the gradient vector and Hessian matrix components. However, as stated by Carnahan et al.(1969) and as experienced by us in previous work, numerical differentiation is an inherently inaccurate process, and particularly for high order differentiation, where errors tend to be magnified still further. Moreover, the optimal step size used in the evaluation of the differentiation formula is difficult to determine.

On the contrary, differentiation via calculus, whenever possible, provides the exact results for partial derivatives of any order. The accuracy in the computations is an important requirement for the QP algorithm to arrive to the exact solution. At the same time, derivation via calculus reduces the computation time significantly. The expressions for the partial derivatives presented in Appendix IV contemplate the general case of a reservoir receiving natural inflows and releases from upstream reservoirs, and in turn, the releases from the referred reservoir affecting downstream water diversions and powerplants. With the set of partial derivatives coded as an external library, the assemblage of the gradient vector and Hessian matrix for any new reservoir system will be highly simplified.

SET OF CONSTRAINT EQUATIONS

Chapter 2 presented a general discussion of the operational constraints that regulate the reservoir system, paying no special attention to the format used for writing the constraint equations. This section, in particular, concentrates on the constraint equations format as required by the quadratic programming algorithm to solve the optimization problem. Subroutine QPMOR, which solves the standard QP problem, accepts as input a set of constraint functions with only equalities

and inequalities of the type [≥], and where all the equalities should be listed first. However, QPMOR imposes no restriction on the sign of the right-hand side terms, as it was the case for the LP/Phase-1 algorithm. What follows is the general format of the constraint equations as implemented during the formulation of the QP problem. The equations listed herein are derived directly from the corresponding equations in Chapter 2, where all the notation is given:

a. Prescribed Final Storage: this is an equality constraint, derived from Eq.(2.17),

$$\sum_{k=1}^{N_p}\left(-T_k^u + D_k^u + T_k\right) = S^{o} - S^{f} + \sum_{k=1}^{N_p}\left(Q_k + L_k^u\right) - \sum_{k=1}^{N_p}\left(L_k + E_k\right) \tag{5.6}$$

b. Capacity Constraint: this is an inequality constraint, derived from Eq.(2.15),

$$\sum_{k=1}^{i}\left(-T_k^u + D_k^u + T_k\right) \geq S^{o} - S_M + \sum_{k=1}^{i-1}\left(Q_k + L_k^u\right) - \sum_{k=1}^{i-1}\left(L_k + E_k\right) \qquad \textit{for } i = 1,2,....,N_p \tag{5.7}$$

c. Release Constraint: this is an inequality constraint, derived from Eq.(2.16),

$$\sum_{k=1}^{i}\left(T_k^u - D_k^u - T_k\right) \geq S_m - S^{o} - \sum_{k=1}^{i-1}\left(Q_k + L_k^u\right) + \sum_{k=1}^{i-1}\left(L_k + E_k\right) \qquad \textit{for } i = 1,2,....,N_p \tag{5.8}$$

d. Minimum Recreation Flow: this is an inequality constraint, derived from Eq.(2.20),

$$T_i - D_i \geq Frec - L_i \qquad \textit{for } i = 1,2,....,N_p \tag{5.9}$$

The list above accounts for only four out of the six constraints listed in Chapter 2. The remaining two constraints are not needed in the QP formulation. The maximum and minimum turbine capacity and maximum and minimum water-supply are automatically accounted for by the upper and lower bounds of the control variables within QPMOR. Furthermore, by imposing the constraint equation that controls the minimum recreational flow in the river, the condition that an off-stream diversion should not exceed the total reservoir release will be simultaneously satisfied. The set of constraints for the present problem, and for a 24 months operational horizon, is made up of a total of 146 equations, 2 equalities plus 144 inequalities.

MODELING OF WATER LOSSES

Notice that in Eqs.(5.6) to (5.9) the unknown variables were moved to the left-hand side of each constraint, while all the known (or assumed known) terms are in the right-hand sides. This statement implies that the model assumes as known beforehand the values for the evaporation losses and the reservoir spills, when solving the QP problem. The fact that evaporation and spills are part of the RHS's, makes the set of QP constraint functions no longer a "hard" set of constraints. Once the QP problem is solved, all RHS terms should be recomputed in order to account for possible changes in the variables T and L making them up. The QP problem is then solved repeatedly in the sequential fashion explained in Chapter 2.

A primary concern during the formulation of the optimization model was to anticipate the way the model would perform in regard to the imperfect modelling of spillway releases and evaporation losses. The optimal solution provided by the QP algorithm after each sequence needs to be **adjusted** in order to find storage values that estimate spills and evaporation losses accurately. Since both outflows are directly proportional to the reservoir storage level, they can be lumped into a single term named **Losses**, see also Eq.(2.5). Then, the question to answer was: ... May an optimal solution (provided by the QP algorithm) that is feasible with respect to the original constraint set become infeasible with respect to the new constraint set ?. That is, with respect to the constraint set computed after adjusting the state of the system due to the new values of the reservoir losses. In order to answer this question, let's analyze the following hypothetical cases, where a storage trajectory tends to approach and depart from the upper and lower storage bounds during the sequential optimization process.

(i) Storage trajectory approaching the bounds: Figure 5.1 shows a storage trajectory of a single reservoir at three different stages during the optimization process. The dotted line represents the optimal trajectory resulting from the last sequence of QP, denoted in the figure by the index $(i-1)^*$. This last trajectory is the one utilized as the initial feasible solution for the new optimization cycle, identified by the index (i). However, for a storage trajectory that is successively moving toward the lower bound (left portion of the figure), the water losses accounted for by the new QP solution (dashed line), are overestimated, since they were based on the preceding state of the system, trajectory $(i-1)^*$, which has higher storage levels. Consequently, after the adjustment of the reservoir losses is carried out, the resulting new optimal trajectory should recede inward, as depicted in the graph by the solid line $(i)^*$.

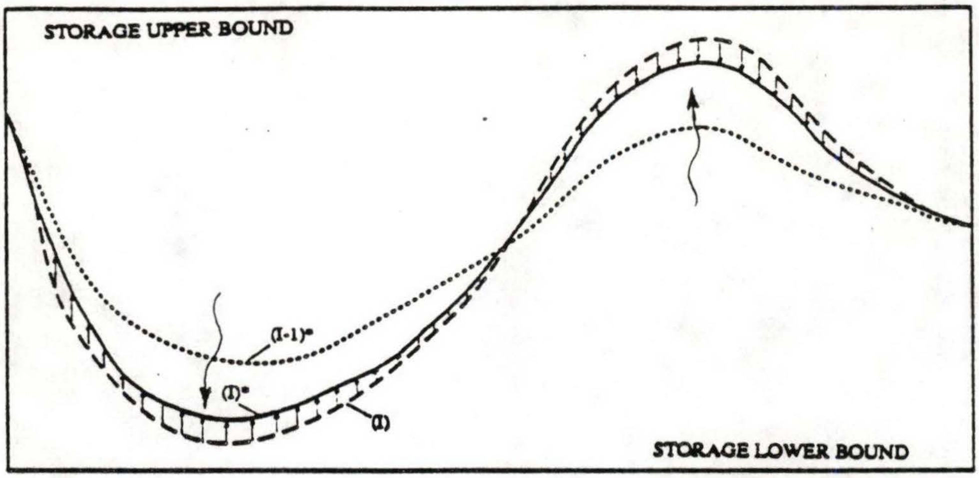

Fig 5.1 Storage Trajectory Approaching the Bounds

By the same reasoning, for the portion of the storage trajectory that moves successively toward the upper bound, losses accounted for by the QP algorithm are underestimated, since storage levels for the trajectory (i-1)* are lower than the ones corresponding to the new trajectory (i). Hence, the correction in the state of the system introduced after adjusting the reservoir losses will force the resulting new optimal trajectory inward, to the position indicated by the solid line (i)*.

(ii) Storage trajectory departing from the bounds: This case represents the opposite situation from the previous case. Now, Figure 5.2 shows the storage trajectory departing from the upper and lower bounds. In the left portion of the figure, where the trajectory is close to the lower bound (dotted line), water losses accounted for by the QP algorithm to generate the new trajectory (i) (dashed line) are underestimated, since they were computed based on the even lower stages of trajectory (i-1)*. After the corresponding adjustment, the final new optimal trajectory (solid line) should move slightly outward.

Conversely, when the trajectory is successively moving away from the upper bound, right portion of Figure 5.2, reservoir losses are always overestimated, causing the adjusted new trajectory (i)* to move slightly outward, laying somewhere between trajectories (i-1)* and (i).

In summary, the analysis of the hypothetical cases above show that the iterative process by which spillages and evaporation losses are accurately evaluated should not provoke infeasible solutions when successively progressing through the optimization cycles.

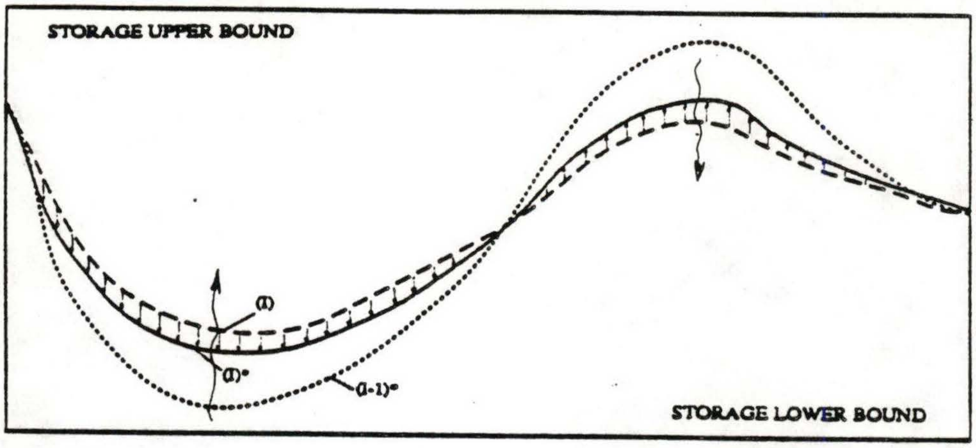

Fig 5.2 Storage Trajectory Departing from the Bounds

There is another important aspect to point out about this approach for indirectly modeling reservoir losses. Although moving all water losses to the RHS's of the constraint functions has the advantage of simplifying the problem formulation, at the same time, we are having less direct control over these variables. When referring in particular to the uncontrolled spills, we should find the way to let the optimization model know that there is such volume of water available for operation, in case that it were economically convenient to store that water for later turbine releases rather than releasing that water instantly through the spillway.

Computationally speaking this is carried out by transforming, previous to solving the QP problem, the reservoir spillage into an 'extra' turbine release during the same period on which the spill takes place. In this manner, the spill volume now becomes part of the optimization scheme, providing the QP algorithm the opportunity to transform totally or partially that volume of water into a useful turbine release at any other time period. When the value of L is added to the value of T, the resulting (fictitious) new turbine release for the period yields a lower marginal benefit for hydropower during that period than what it would have otherwise. Consequently, the objective function drives the model toward a different (and improved) solution, where the excess of water during that particular period is reallocated over other periods of the operational horizon.

GENERAL DESCRIPTION OF MODULE-3

After the task in Module-2 is completed, the feasible solution (though not optimal) provided by the LP/Phase-1 approach will serve as the initial feasible solution required in Module-3 by the Sequential Quadratic Programming (SQP) algorithm. As depicted by the tree-diagram in Figure 5.3, Module-3 is made up of a total of 16 different subprograms. In essence, Module-3 carries out the following tasks:

☞ read in an input-file containing information related to the economic conditions under which the reservoir system operates.

☞ solve a sequence of quadratic programming problems until the maximum value of the objective is reached.

What follows, is a general description of the main steps implemented within Module-3 in order to reach the optimal solution.

```
|
|------ READF3
|------ AGGREGA
|------ OBJECT
|------ QPLHS
|------ LCONVERT
|------ CUMULA
|------ QPRHS
|------ TAYLOR
|------ XSET_XQP
|------ QUAFORM
|------ QPSET
|------ QPMOR
|          |------
|          |------
|          |------
|          '------
|------ XQP_XSET
|------ STATE
|------ EVAPO
|------ QPSPILL
|
```

Fig 5.3 Tree-diagram of Module-3

A. <u>Input of Economic-Information</u>: At the top of Module-3, the program reads a data file named <ECONOM.DAT> by means of subroutine **READF3**. An example of file ECONOM.DAT can

be found in Appendix III, which contains the following information: at level 1 (1-->), the coefficients of the linear energy rate functions for powerplants A and C respectively. Level 2 provides the coefficients for the linear models that represent marginal benefits stemming from hydrogeneration. Finally, at level 3, two sets of 12 pairs of coefficients provide the exponentially-decreasing marginal benefits derived from water supplied at demand zones A and C. The user can input different parameters for each month in order to account for seasonal variations in demand.
B. Initial Objective Function Value: As a reference value, subroutine **OBJECT**, which contains the coded objective function to be maximized, computes the objective value that corresponds to the initial feasible solution. The total return is saved in a variable named 'Fold'. For subroutine OBJECT to be able to carry out the computations properly, another subroutine, **AGGREGA**, should be called first. The latter is just an auxiliary subprogram in the sense that it manipulates existing information with the purpose of facilitating and expediting the computations within OBJECT and another subroutine cited later.
C. Computation of the Constraint Matrix: The constrained optimization problem being solved requires the computation of an extensive set of constraint functions. Instead of the traditional approach of reading the constraint matrix from an external file, the model itself generates the matrix coefficients by means of two subroutines, one subprogram for developing the left-hand side coefficients, and another subprogram for computing the right-hand side terms. The automation in the computations of the constraint coefficients has several practical advantages. It saves the user of the model from the tedious and prone to error procedure of generating the aforementioned data file, and at the same time, it avoids the permanent storage of the file on disk.

Constraints are computed and grouped as separate blocks according to the reservoir subsystem they belong to, i.e., reservoir A or C, and to the purpose they serve, per example, to limit maximum storage, to limit maximum discharge, etc. Then, the whole constraint matrix is the result of aggregating all these independent blocks of constraints. This approach provides large flexibility when formulating the QP problem. **QPLHS** is the subprogram used for computing the left-hand side coefficients of the constraint equations. As it can be inferred from Eqs.(5.6) to (5.9), these coefficients remain unchanged during all the optimization cycles. For that reason, subroutine QPLHS is called only once and before entering the sequential solution of the QP problem (SQP).

Beginning of the Sequential Optimization Cycle

D. <u>Incorporation of Spills</u>: At the top of the optimization cycles, before any other subroutine is called, the main program calls subroutine **LCONVERT**. The purpose of this subprogram is to convert any reservoir spill contained in the initial solution (for the sequence) into a fictitious turbine release, which is added to the actual turbine release. A detailed justification of this procedure has been presented in the preceding section.

E. <u>Computation of Right-Hand Side Terms</u>: The right-hand side terms of the constraint equations are computed by subroutine **QPRHS**. Contrary to QPLHS, QPRHS needs to be called repeatedly, before each sequence of QP is performed, since the expressions for the right-hand side terms are functions of variables that can change from one sequence to the next one. Subroutine QPRHS also requires that an auxiliary subprogram, subroutine **CUMULA**, to be run previously. AGGREGA creates auxiliary arrays that are repeatedly used during the computations carried out by QPRHS.

F. <u>Develop Quadratic Approximation</u>: The nonlinear nonquadratic objective function is approximated by a second order function before calling the subprogram that solves the QP problem. This task is performed in three steps:

First step: the main program calls subroutine **TAYLOR**, which computes first and second order partial derivatives of the original objective function with respect to the control variables. All partial derivatives are computed via calculus, about the initial feasible solution for the sequence. As in the case of subroutine OBJECT, subroutine TAYLOR also requires running subroutine AGGREGA a priori.

Second step: before actually computing the standard quadratic form (third step), subroutine **XSET_XQP** should be called. This subprogram aggregates the four arrays containing turbine releases TA, TC, and off-stream diversions DA, DC, into a single control vector $\mathbf{x}^o$ used by the standard quadratic form and the QP routine.

Third step: all partial derivatives and the control vector $\mathbf{x}^o$ are passed to subroutine **QUAFORM**, which computes the first and second order coefficients of the standard quadratic form that will represent, during the optimization process, the original nonquadratic objective function.

G. <u>Set-up Quadratic Programming Problem</u>: This portion of the model, represented by subroutine **QPSET**, provides the link between the quadratic programming routine and the rest of the model. All the information required to solve the QP problem is provided by QPSET exclusively. We should mention: control and output parameters, parameters

defining the QP problem dimensions, type of control variables, upper and lower bounds of variables, and finally, the set of constraint equations restricting the operation of the system. Although not included at this time, subroutine QPSET can eventually be modified in the future to allow the interaction of the user with the model in a dynamic fashion. In other words, either the user (acting externally from the screen), or the model itself (following some decision rules), may have the possibility to interact with the model by modifying the set of constraints that the QP maximization problem is subject to. The specification of constraints as separate blocks (see level C) should facilitate this task. This approach has the potential to drastically reduce the execution time of the model.

H. Solving the Quadratic Programming Problem: An efficient quadratic programming algorithm is a basic requirement for the successful operation of the SQP technique. Subroutine **QPMOR**, developed by Jønch-Clausen and Morel-Seytoux (1977) and modified by Leifsson and Morel-Seytoux(1981) is the one used. Subroutine QPMOR encompasses a large series of subprograms that solve a convex quadratic programming problem using an algorithm known as the General Differential Algorithm, first presented by Wilde & Beightler (1967) and developed in more computational details by Morel-Seytoux (1972). Subroutine QPMOR receives the initial feasible solution, vector $\mathbf{x}^o$, and returns to the main program a set of new values for the control variables represented by the vector $\mathbf{x}$. The returning vector $\mathbf{x}$ is then desegregated by subroutine **XQP_SET** into its four components TA , DA , TC , and DC . The renaming of the control variables performed by XQP_XSET, though not strictly necessary, facilitates the interpretation of the code.

I. Computation of the State of the System: After QPMOR, the resulting state of the system is computed by means of subroutine **STATE**. The latter subroutine computes the new reservoir storage trajectories based on all inflows and outflows from the reservoirs. The auxiliary subroutine CUMULA should be called prior to STATE to precompute arrays that contain the inflows and outflows in the form required by subroutine STATE.

J. Computation of Water Losses: The indirect approach used by the model to account for evaporation and reservoir spills require the recomputation of these two terms after solving each QP problem. The computations are carried out as follows:

Evaporation: The new storage levels at the reservoirs, after subroutine STATE, is the information utilized by subroutine **EVAPO** to recompute net evaporation losses, based on the new reservoirs surface area exposed to evaporation. A change in evaporation (usually

a small quantity) at any time period, is algebraically added to the turbine release that corresponds to that same period. In this manner, mass balance is preserved.

Reservoir Spills: The remaining water loss, reservoir spillages, is detected and computed by subroutine **QPSPILL**. This subroutine checks for spills that could have been masked by the QPMOR solution in the form of turbine release that exceed penstock capacity. In other words, if the value of the control variable associated with a turbine release is larger than penstock capacity, QPSPILL automatically transforms the excess flow into a reservoir spill, leaving turbine release equal to penstock capacity.

K. Compute Objective Value at New Solution: Finally, the value of the objective function at the new feasible (and improved) solution is computed by calling subroutine OBJECT (and previously AGGREGA). The new total benefit is stored in a variable named 'Fnew'.

End of Sequential Optimization Cycle

L. ANOTHER QP SEQUENCE ? At the end of each optimization cycle the main program compares the objective function value after the current sequence 'Fnew', with the value from the preceding sequence 'Fold'. If the change in the objective value during per example five consecutive sequences is less than a stipulated tolerance, it is assumed that the optimal solution has been reached and the program stops. Otherwise, a new optimization sequence is required, the variable 'Fold' takes the value of 'Fnew' and the main program returns to the level D indicated above for a new optimization cycle.

TESTING OF MODULE-3

This section presents a numerical example that illustrates the optimization of the reservoir system as carried out by Module-3. The initial feasible solution obtained from the LP/Phase-1 approach in the previous chapter has been altered in order to more fully appreciate the potential of Module-3 as an optimization tool. Looking at Table 4.3, it can be seen that turbine releases from both powerplants for almost all time periods are at their maximum capacity, leaving practically no room for the optimization algorithm to search for a more optimal water allocation pattern. Therefore, besides changes in volume and time period in which reservoir spills occur, the maximum turbine discharges at both powerplants were increased. This last change provides more space for the optimization model to operate. Figure 5.4 sketches maximum and minimum storage levels and turbine releases for the two reservoirs as utilized by Module-3, which can be compared with the values shown in Figure 4.3 used for testing Module-2.

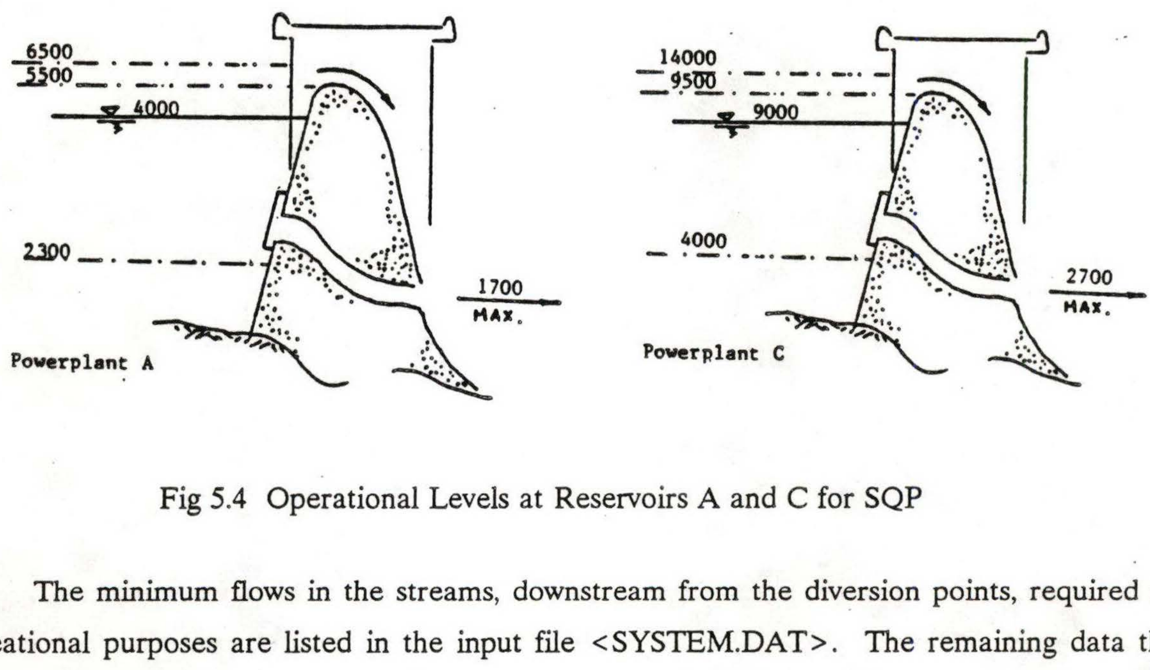

Fig 5.4 Operational Levels at Reservoirs A and C for SQP

The minimum flows in the streams, downstream from the diversion points, required for recreational purposes are listed in the input file <SYSTEM.DAT>. The remaining data that complete the input necessary to run Module-3 are provided by the input file <ECONOM.DAT>, included in Appendix III. The off-stream water to be allocated to the demand zones should satisfy the following seasonal characteristics: (a) the winter season, where only urban water users are present, (b) spring and fall season, with urban users and irrigation at a moderate level of consumption, and (c) summer season, similar to the previous season but with a higher comsumption

of water by the irrigation users. This seasonal distribution is totally arbitrary, and is presented only for the purpose of the test problem. The three seasons are represented in the model by three different exponential decreasing benefit curves as depicted in Figure 5.5, and where for the sake of comparisson, the marginal benefit curve for hydropower was also included. Notice how as we move into the summer, when the water demand is higher, the corresponding demand curves extend further in the axis of abscissas. Also notice that the three curves reach the axis of ordinates at three different levels, 80000., 50000., and 25000., for curves a), b), and c), respectively. In a real case, unless there were a special reason for changing the demand curve for urban-water from season to season, the three curves on Figure 5.5 would start from the same ordinate level.

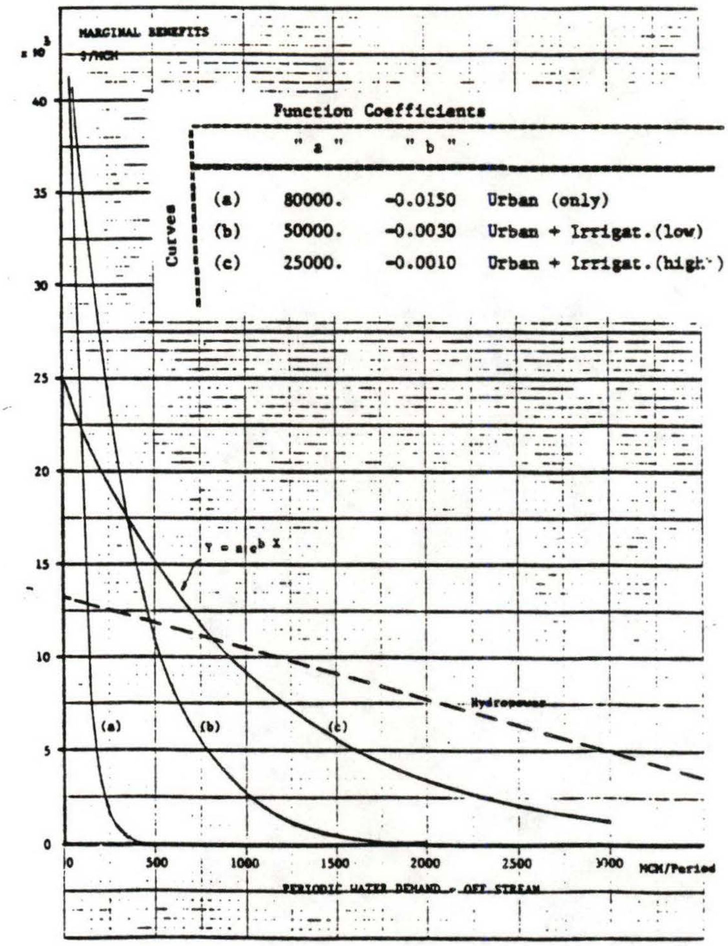

Fig 5.5 Seasonal Water Demand Curves

Figures 5.6 and 5.7 display the initial (suboptimal) and final (optimal) solutions for the reservoir operation problem respectively. Several time series were plotted in these figures, corresponding to reservoir storage, turbine releases, reservoir spills, and water diversions for the two reservoir subsystems. The operational horizon corresponds to 24 months. The four windows on the left side correspond to reservoir A, while the other four to the right belong to reservoir C. All bar diagrams were scaled to the same volume level in order to admit comparisons among them. The other two variables of interest, net evaporation and recreation flows were not plotted.

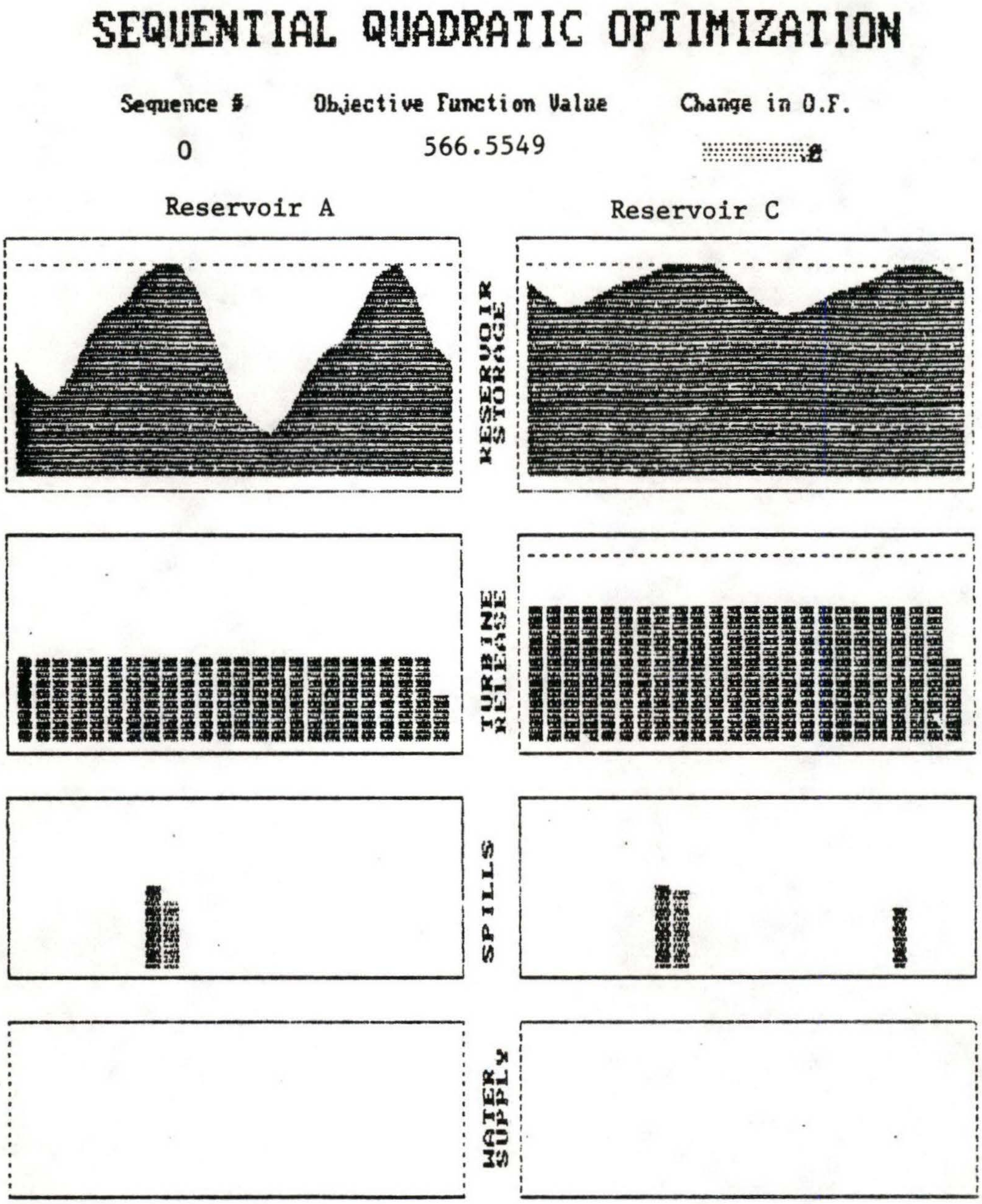

Fig 5.6 Initial Feasible Solution - Graphical Display

Nevertheless, they were listed in Tables 5.1 and 5.2 together with the rest of the variables, representing the input and output from Module-3 respectively.

The initial feasible solution, Table 5.1 and Figure 5.6, show equal turbine releases from both powerplants for practically all time periods, with the reservoirs showing some degree of regulation to accomodate the natural inflows. Reservoir A spills during periods 8 and 9, while the downstream reservoir spills during three periods, 8, 9, and 21. Also notice that no water is diverted to any of the demand zones. Obviously, although feasible, this initial solution is far from being optimal in an economical sense. Not only because no water diversion means loss of benefits, but also because the volume of water that is spilled may otherwise have the oportunity to be stored in the reservoirs and generate benefits later on.

Table 5.1 Initial Feasible Solution - Numerical Table

	Reservoir A							Reservoir C						
Period	Q	E	T	L	D	REC	S	Q	E	T	L	D	REC	S
							4000.0							9000.0
1	900.	29.0	1232.0	.0	.0	1232.0	3639.0	400.	56.0	1950.0	.0	.0	1950.0	8626.0
2	1100.	25.0	1232.0	.0	.0	1232.0	3482.0	500.	46.0	1950.0	.0	.0	1950.0	8362.0
3	1600.	23.0	1232.0	.0	.0	1232.0	3827.0	800.	39.0	1950.0	.0	.0	1950.0	8405.0
4	1800.	12.0	1232.0	.0	.0	1232.0	4383.0	1000.	23.0	1950.0	.0	.0	1950.0	8664.0
5	1600.	12.0	1232.0	.0	.0	1232.0	4739.0	1000.	16.0	1950.0	.0	.0	1950.0	8930.0
6	1400.	12.0	1232.0	.0	.0	1232.0	4895.0	900.	16.0	1950.0	.0	.0	1950.0	9096.0
7	1700.	11.0	1232.0	.0	.0	1232.0	5352.0	900.	14.0	1950.0	.0	.0	1950.0	9264.0
8	2800.	11.0	1232.0	1409.0	.0	2641.0	5500.0	1000.	16.0	1950.0	1439.0	.0	3389.0	9500.0
9	2400.	12.0	1232.0	1156.0	.0	2388.0	5500.0	900.	21.0	1950.0	1317.0	.0	3267.0	9500.0
10	800.	20.0	1232.0	.0	.0	1232.0	5048.0	700.	35.0	1950.0	.0	.0	1950.0	9447.0
11	500.	21.0	1232.0	.0	.0	1232.0	4295.0	500.	44.0	1950.0	.0	.0	1950.0	9185.0
12	400.	30.0	1232.0	.0	.0	1232.0	3433.0	400.	56.0	1950.0	.0	.0	1950.0	8811.0
13	900.	29.0	1232.0	.0	.0	1232.0	3072.0	400.	56.0	1950.0	.0	.0	1950.0	8437.0
14	1100.	25.0	1232.0	.0	.0	1232.0	2915.0	500.	46.0	1950.0	.0	.0	1950.0	8173.0
15	1600.	23.0	1232.0	.0	.0	1232.0	3260.0	800.	39.0	1950.0	.0	.0	1950.0	8216.0
16	1800.	12.0	1232.0	.0	.0	1232.0	3816.0	1000.	23.0	1950.0	.0	.0	1950.0	8475.0
17	1600.	12.0	1232.0	.0	.0	1232.0	4172.0	1000.	16.0	1950.0	.0	.0	1950.0	8741.0
18	1400.	12.0	1232.0	.0	.0	1232.0	4328.0	900.	16.0	1950.0	.0	.0	1950.0	8907.0
19	1700.	11.0	1232.0	.0	.0	1232.0	4785.0	900.	14.0	1950.0	.0	.0	1950.0	9075.0
20	1800.	11.0	1232.0	.0	.0	1232.0	5342.0	1000.	16.0	1950.0	.0	.0	1950.0	9341.0
21	1400.	12.0	1232.0	.0	.0	1232.0	5498.0	1900.	21.0	1950.0	1002.0	.0	2952.0	9500.0
22	800.	20.0	1232.0	.0	.0	1232.0	5046.0	700.	35.0	1950.0	.0	.0	1950.0	9447.0
23	500.	21.0	1232.0	.0	.0	1232.0	4293.0	500.	44.0	1950.0	.0	.0	1950.0	9185.0
24	400.	30.0	663.0	.0	.0	663.0	4000.0	400.	56.0	1192.0	.0	.0	1192.0	9000.0

Now, let's analyze Table 5.2 and Figure 5.7, which display the optimal solution. Both reservoirs were able to accomodate most of the volume originally spilled, in fact, spills were completely eliminated except for the 9th period at the upstream reservoir. This is achieved by increasing turbine releases prior to the period in which the spills occur, consequently creating the empty storage that permits to store almost all inflows. The reason why part of the original spill still

Table 5.2 Optimal Solution - Numerical Table

	Reservoir A							Reservoir C						
Per.	Q	E	T	L	D	REC	S	Q	E	T	L	D	REC	S
							4000.0							9000.0
1	900.	26.2	1083.0	.0	883.0	200.0	3790.8	400.	43.4	2070.6	.0	1570.6	500.0	7485.9
2	1100.	24.4	1139.0	.0	939.1	199.9	3727.4	500.	37.5	2132.0	.0	1631.9	500.1	6016.3
3	1600.	22.4	1136.7	.0	575.3	561.4	4168.2	800.	29.3	1373.4	.0	973.3	400.0	5975.1
4	1800.	12.6	1327.9	.0	597.2	730.7	4627.7	1000.	17.4	1453.9	.0	1053.9	400.0	6234.5
5	1600.	11.6	1541.9	.0	154.6	1387.3	4674.2	1000.	13.4	912.9	.0	610.9	302.0	7695.5
6	1400.	14.2	1700.0	.0	158.8	1541.2	4360.0	900.	15.1	1414.0	.0	539.5	874.5	8707.6
7	1700.	11.2	1700.0	.0	164.7	1535.4	4348.8	900.	13.9	1860.0	.0	600.0	1260.0	9269.0
8	2800.	11.9	1700.0	.0	172.3	1527.7	5436.9	1000.	16.7	2279.9	.0	600.0	1679.9	9500.0
9	2400.	14.3	1700.0	622.6	721.5	1601.1	5500.0	900.	21.7	2479.4	.0	2079.4	400.0	9500.0
10	800.	23.6	977.0	.0	469.0	508.0	5299.4	700.	36.1	1171.8	.0	772.2	399.6	9500.0
11	500.	24.1	1051.6	.0	740.1	311.6	4723.6	500.	43.8	1855.6	.0	1355.6	500.0	8412.1
12	400.	35.1	986.6	.0	786.7	199.9	4101.9	400.	51.8	1906.5	.0	1406.4	500.1	7053.7
13	900.	26.7	1020.7	.0	822.0	198.7	3954.6	400.	35.9	1957.2	.0	1457.2	500.0	5659.2
14	1100.	25.1	1073.5	.0	874.7	198.8	3955.9	500.	30.1	2012.5	.0	1512.5	500.0	4315.5
15	1600.	23.5	913.9	.0	565.2	348.7	4618.6	800.	22.7	1278.9	.0	878.9	400.0	4162.5
16	1800.	13.5	1132.8	.0	584.7	548.1	5272.3	1000.	13.4	1335.6	.0	935.5	400.1	4361.7
17	1600.	12.6	1359.7	.0	151.4	1208.3	5500.0	1000.	10.8	608.9	.0	308.8	300.1	5950.3
18	1400.	15.8	1488.4	.0	154.1	1334.4	5395.7	900.	13.0	811.2	.0	511.2	300.0	7360.5
19	1700.	12.6	1670.5	.0	157.7	1512.8	5412.7	900.	12.6	1303.7	.0	515.0	788.8	8456.9
20	1800.	12.7	1700.0	.0	163.1	1536.9	5500.0	1000.	16.0	1767.2	.0	600.0	1167.2	9210.7
21	1400.	14.3	1385.7	.0	697.6	688.1	5500.0	1900.	21.4	2277.3	.0	1877.2	400.1	9500.0
22	800.	23.8	776.2	.0	409.9	366.3	5500.0	700.	36.1	1030.1	.0	630.8	399.4	9500.0
23	500.	24.5	1188.5	.0	423.4	765.1	4787.0	500.	45.5	1356.7	.0	858.4	498.3	9362.9
24	400.	35.0	1152.0	.0	455.0	696.9	4000.0	400.	58.8	1401.0	.0	902.6	498.5	9000.0

remains as such in the optimal solution is attributed to the not always easy to predict balance among all the hydropower ingredients embedded in the optimization model: turbine release, reservoir head, and the decreasing marginal benefits for energy production. In order to verify that this state of the system was the optimal one, the model was forced to transform absolutely all

uncontrolled spills into turbine releases by means of penalty factors. Under this new condition, the total economic benefit decreased, proving that the solution provided by the model was actually the optimal solution. The decrease in total return occured because of the earlier depletion of the storage that had to take place in order to accomodate the complete series of inflows. The resulting

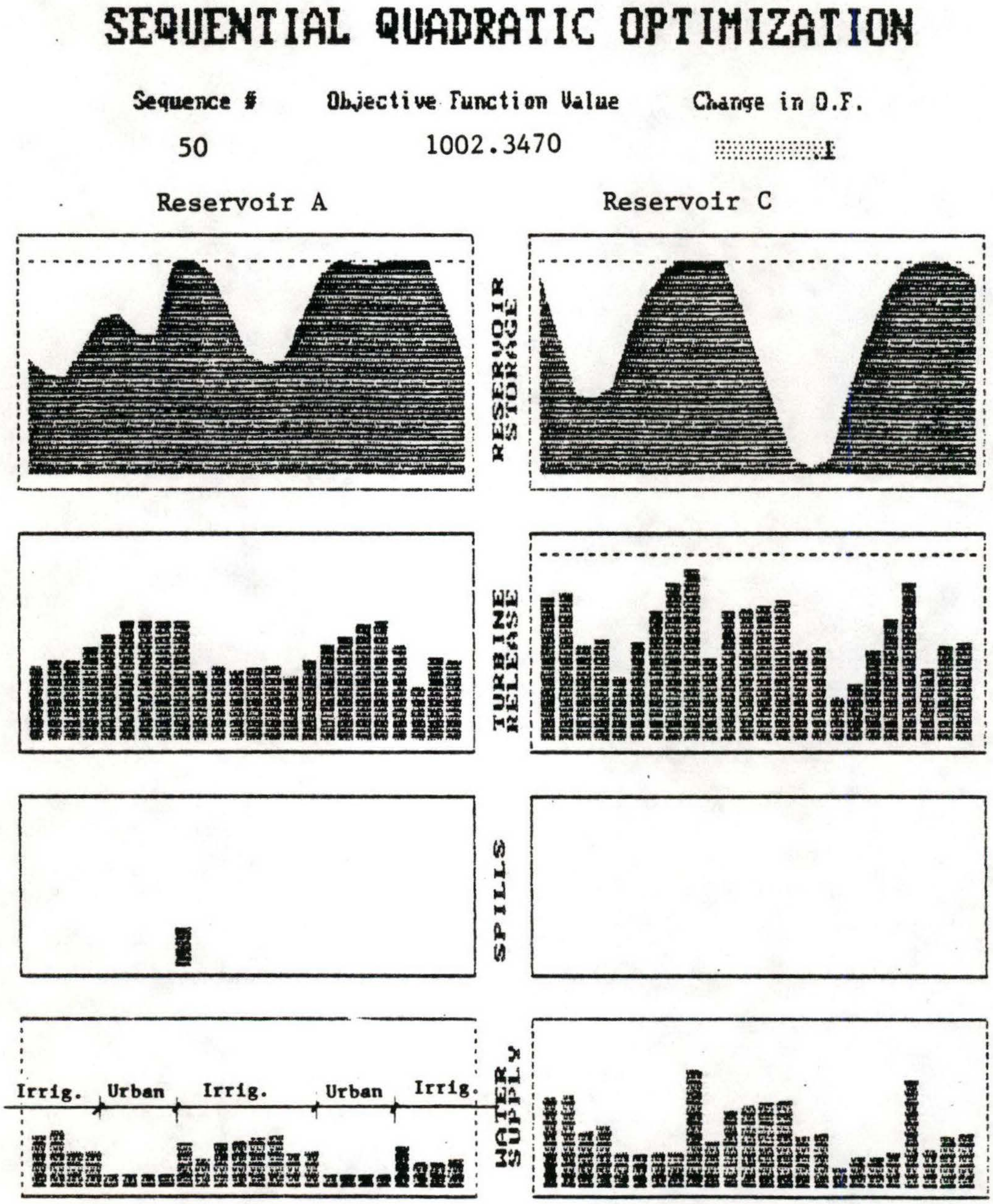

Fig 5.7 Optimal Solution - Graphical Display

loss in energy generation during the earlier periods (powerplants generating at lower heads), is not compensated by the extra volume of water that flows through the turbines.

An interesting feature of this optimal water allocation problem is to observe the trade-off that the model exerts between competing water users. The more evident trade-off takes place for each unit of water that after being released from reservoir A can either be diverted to the demand zone A or be directly routed to the second reservoir for hydropower production. The decision at each period is mainly controlled by the willingness to pay that each one of the users has for the water, which is reflected by the marginal price functions for water-suppply and hydropower respectively. Several water supply marginal benefit curves were tested until finding a set of curves that roughly diverts one third of the total outflow from reservoir A into demand zone A, while the remainning two thirds enter reservoir C.

The two windows at the bottom of Figure 5.7 show how the model allocates water to the upstream and downstrean demand zones. The seasonal pattern for water demand, low flows during the winter months and higher flows during the irrigation season, comes out as a result of specifying different marginal benefit curves for different seasons of the year. For example, looking at the demand zone A in Table 5.2, from the 5^{th} to the 8^{th} period of each year, the water allocation is around 155 MCM, serving only municipal and industrial users. During the rest of the year, the irrigation demand is added, reaching a peaking supply during the 1^{st}, 2^{nd}, 11^{th}, and the 12^{th} period of the year, with flows in the neighborhood of 850 MCM.

Demand zone C is a more singular case, since not having a reservoir downstream that competes for the same water (only minimum recreational-flow requirements), more water is allocated to this area in comparisson to zone A. Besides, it would be unlikely that the model would assign 'exactly' the same flow level during periods that share the same marginal benefits for water-supply. Differences of this kind will be more accentuated for wet inflow series. Once the recreational flow is satisfied, any remainning flow is diverted for consumptive use, as it happens during the 9^{th} period, which sticks out from the general pattern. This differences become more accentuated the wetter the inflow series is.

Very minor differences can be found between the minimun recreational flows specified, and the flows that are actually shown in Table 5.2. See per example periods 23 or 24 at the most downstream portion of the system. While a minimum of 500 MCM/month is required, the model found values of 498.3 and 498.5 MCM respectively. This slight difference is due to the effect that the sequential computation of evaporation losses has on the right-hand side of the constraints that

control the minimum recreational flows.

Module-3 was run, independently from the previous modules, in a personal computer with a 80386 Intel processor running at 25 Mhz. The program was compiled using the Microsoft-Fortran compiler version 2.3, taking practically all the memory addressable by the operating system DOS. The total execution time for 50 QP sequences was approximately 25 minutes, with an average time of 25 seconds per QP sequence. Figure 5.8 displays the evolution of the objective function value as a function of the computation time, and where each symbol in the curve represents a new QP sequence.

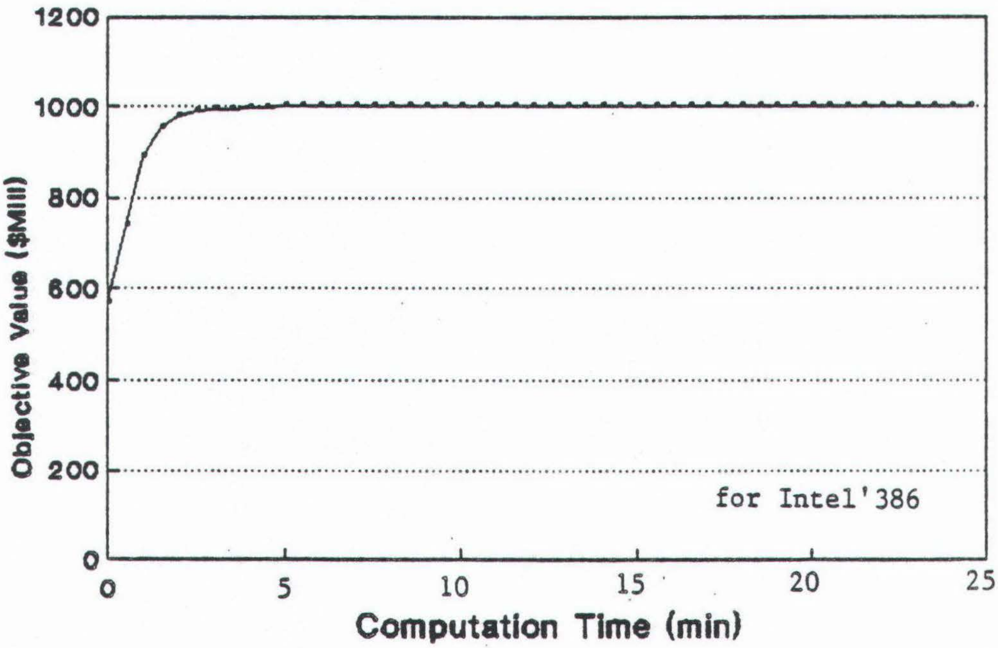

Fig 5.8 Evolution of the Objective Function Value

The total computation time for Module-3 will vary depending on the type of hydrology affecting the operation of the system. The example above can be considered typical for inflows during relatively wet years. A more average hydrologic condition will demand a shorter computation time. Furthermore, provided that the maximum number of QP sequences specified does not abort the execution, equal to 50 in this example, the algorithm will continue operating until the prescribed accuracy level is reached. Obviously, the greater the accuracy level the longer the computation time. For this example, the accuracy tolerance was equal to 0.0005 per cent of change

in the value of the objective function. This small value explains the long and flat tail shown by the objective value in Figure 5.8. Another factor that affects the computation time, although less significantly, is the number of QP levels specified per sequence of SQP. For the problem at hand, it is recommended to select any number between 20 to 40 levels of QP per sequence of SQP, 30 was chosen for this example.

Chapter 6

MODEL APPLICATION

STOCHASTIC OPTIMIZATION

During the second stage of this study, optimum feedback operating policies will be derived. This approach, also known as "implicit stochastic optimization", involves application of a multivariate regression model to results of a set of deterministic optimization runs.

The deterministic runs would be done for several hydrologic traces for each stochastic regime of interest, and would provide a benchmark indicating the most efficient allocation possible according to the economic demands and constraints specified. For the same monthly inflow series with the addition of the flow increase in the region where such increase can be generated, the optimization will be repeated. The difference in total benefits between the two runs (without and with increase in flows) is a measure of the value of the increase, although under optimistic conditions (deterministic future). However, since the scenario is optimistic in both cases (without and with increase in flow), the difference is probably valid for realistic conditions of operations. Repetition of this pair of runs for several traces of flows will provide a distribution of the economic value of the flow increases for which mean, variance and skewness can be calculated.

The deterministic runs will provide the data necessary to perform regression analysis to predict the "marginal value of storage" given the state of the system. The following step in the study will consist on deriving a set of operational rules for the reservoir system. They will consist of a body of decision rules using current values of the state variables (and perhaps short-term inflow forecast) to decide whether inflow should be stored or released.

OPERATIONAL RULES

The objective of the operational rules module is to describe the disposition of flow increases as a function of the characteristics of inflows, water management facilities, and demands. Variation in preincrease inflows will be modeled by using flow records from numerous western rivers, while variation in flow/increase is simply created by varying the percent increase. Variation in storage facilities can be simulated by varying the ratio of storage to inflow at the two reservoirs. Variation in electricity production can be simulated with different turbine capacities. Variation in demand

can be simulated by altering the value of hydroelectric energy production, the volume and value of diversions, and the instream flow constraint.

Since the optimization is carried out in a monthly time step, operational rules for each month can be deduced. Each reservoir of the system will have associated a set of twelve (12) regression equations, one for each month. A unique regression equation per reservoir is not recommended due to the expected seasonality on the regression coefficients.

There is not a unique regression function which may serve the purpose. In fact, there would be several combinations of variables that could be used to derive operational rules for operating the system in real-time. The dependent variable will be selected for the purpose of prediction, being defined by the problem to be solved. The selection of the independent variables should be such that they have been observed in the past concurrently with the dependent variable, so that the prediction equation may be established, and they continue to be observed or estimated in the future, so the independent variable may be predicted from them when necessary.

Typical optimal feedback policies may permit selection of end-of-period target storage levels based on the state of the system. Other functions may use releases during the period as the dependent variable, with the independent variables such as storage at beginning of the month, inflow during the month, previous inflow (previous month or two; an indicator of wetness) or maybe better, storage still, but deviation of inflow during month from "normal" value, etc.

Finally, the analyst will establish a multiple regression equation between the **marginal value of the release** for the month and the predictive variables. Two sets of operational rules will be deduced, one set for natural flow conditions (natural system rules) and a second set for inflows carrying small increases in streamflow (increase-flow rules). Both sets of operational rules will have embedded the most favorable conditions of operations, without and with flow-increases respectively.

REAL TIME SIMULATION

The last stage of the study will use the derived decision rules to optimize the operation of the system by simulation. The purpose is to operate the system in real time for the same hydrologic traces that were used for the deterministic runs. Two new distributions of the value of flow increases will be derived under the following scenarios:

a) using natural system rules for both natural flows and increased flows,

b) using natural system rules for natural flows and increase-flow rules for increased flows. One thus measures the value of the flow increases under realistic uncertain conditions of flow,

without and with institutional (operational) changes.

In summary, for every pair of runs one measures the value of the flow increases under realistic uncertain conditions of flow but without institutional (operational) changes to reflect the existence of the augmented flows. All that values generated will provide a distribution for that economic measure. The analyst can repeat the runs under conditions of flow increases but this time using the new rules that account for the recognized existence of flow increases. The difference between total benefits for each "augmented" run and the corresponding reference run will provide a measure of the value of flow increase.

Comparison of the deterministic results (i.e., the disposition of the flow increases given the deterministic optimizations) with the results of the stochastic runs will indicate the difference between the ideal (the benchmark) and the more realistic situation. We may find that there is little difference between these two approaches, because in each case the variable of interest is a difference between with and without flow increase runs. And, comparison of the stochastic run results given only the "without" rules, with the stochastic run results given "with" rules used for the "with" flows and the "without" rules used for the "without" flows, will indicate the importance of changing reservoir operating rules to deal with flow changes.

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APPENDIX I

< HYDRO.DAT >

Input File for Module-1

```
*************************************
*        H Y D R O . D A T          *
*************************************
```

1-->

TRANSF	INDEP	AUTREG	INCFLO
> TRUE	> TRUE	>(2)	> FALSE

Relative Area of the Basins ARE_

2-->

BASIN A	BASIN B
> 1.0	> 1.0

Select Unit-System for all Flow Records

3-->
UNSYS > CMS

Flow	Volume
CFS	A-F
CMS	MCM

Historic Mean-Annual Flows HMAF_

4-->

River A	River B
> 425.3	> 284.9

RIVER at BASIN A ... 39 years of record
Historical Monthly Flows ... HMF_(NR,NM)

5-->

1	2	3	4	5	6	7	8	9	10	11	12
90.	190.	412.	571.	636.	614.	548.	642.	762.	318.	139.	114.
130.	234.	260.	261.	348.	518.	650.	673.	340.	142.	98.	78.
68.	145.	151.	322.	208.	376.	503.	351.	175.	76.	45.	39.
33.	82.	817.	1197.	735.	645.	978.	1001.	656.	259.	201.	149.
215.	900.	1420.	775.	855.	877.	1023.	961.	649.	297.	254.	207.
130.	196.	215.	515.	532.	749.	538.	695.	588.	301.	144.	80.
63.	212.	643.	470.	381.	373.	562.	509.	259.	154.	83.	50.
59.	118.	367.	608.	336.	676.	930.	801.	450.	220.	143.	384.
242.	1116.	1766.	628.	369.	320.	437.	366.	224.	153.	77.	63.
111.	661.	986.	608.	783.	700.	639.	910.	846.	530.	379.	210.
124.	940.	1531.	1149.	940.	774.	791.	810.	607.	278.	118.	164.
110.	364.	297.	404.	319.	372.	397.	249.	124.	130.	81.	60.
55.	215.	505.	474.	846.	638.	507.	939.	745.	223.	88.	56.
104.	196.	300.	433.	1150.	679.	783.	980.	733.	253.	162.	103.
156.	260.	306.	364.	341.	367.	507.	475.	244.	609.	157.	130.
212.	411.	271.	364.	425.	352.	456.	496.	195.	91.	63.	50.
36.	150.	248.	540.	675.	493.	626.	791.	611.	233.	93.	39.
29.	449.	822.	1456.	726.	417.	568.	484.	239.	112.	80.	54.
320.	511.	595.	743.	737.	806.	836.	830.	411.	190.	72.	45.
66.	127.	407.	578.	632.	435.	771.	748.	403.	289.	175.	75.
131.	226.	632.	862.	747.	629.	838.	727.	352.	174.	86.	57.
45.	76.	148.	264.	608.	650.	390.	303.	180.	83.	52.	46.
119.	223.	381.	436.	452.	1029.	928.	953.	561.	251.	167.	115.
99.	164.	347.	233.	300.	617.	693.	614.	513.	276.	201.	162.
165.	342.	1119.	741.	928.	428.	612.	822.	897.	303.	114.	108.
205.	422.	556.	789.	409.	276.	493.	696.	813.	355.	197.	108.

98.	296.	287.	353.	524.	537.	822.	862.	522.	208.	127.	127.
97.	129.	165.	245.	305.	511.	416.	392.	285.	180.	95.	60.
63.	302.	578.	743.	963.	602.	637.	751.	572.	239.	123.	76.
84.	195.	444.	481.	562.	467.	545.	648.	539.	360.	286.	144.
121.	459.	418.	953.	914.	600.	672.	744.	534.	348.	181.	99.
75.	524.	1110.	727.	1226.	592.	532.	695.	423.	235.	124.	87.
73.	152.	588.	622.	457.	406.	625.	656.	375.	301.	180.	95.
67.	183.	317.	347.	585.	466.	717.	636.	431.	225.	160.	113.
140.	289.	428.	563.	421.	452.	604.	842.	624.	364.	182.	81.
74.	85.	900.	591.	369.	383.	542.	489.	342.	226.	123.	86.
79.	450.	829.	705.	592.	678.	876.	804.	573.	236.	118.	76.
57.	147.	369.	1264.	738.	593.	832.	776.	351.	138.	79.	69.
48.	145.	233.	290.	1392.	1009.	621.	728.	732.	226.	160.	159.

RIVER at BASIN B ... 39 years of record
Historical Monthly Flows ... HMF_(NR,NM)

6-->

1	2	3	4	5	6	7	8	9	10	11	12
108.	126.	193.	249.	498.	480.	332.	380.	461.	387.	286.	212.
172.	199.	244.	202.	202.	244.	290.	336.	259.	178.	115.	84.
69.	119.	151.	256.	205.	206.	262.	241.	190.	126.	82.	50.
48.	81.	349.	552.	482.	444.	524.	627.	586.	370.	249.	170.
183.	423.	635.	505.	488.	482.	478.	505.	472.	335.	267.	200.
133.	165.	179.	235.	280.	373.	317.	407.	421.	324.	238.	163.
96.	188.	346.	326.	243.	220.	247.	267.	224.	189.	125.	82.
69.	102.	227.	324.	254.	300.	361.	365.	337.	281.	223.	349.
267.	494.	752.	535.	331.	232.	266.	278.	204.	149.	94.	73.
85.	219.	383.	354.	366.	400.	359.	402.	438.	403.	326.	219.
147.	397.	729.	629.	554.	453.	410.	413.	389.	280.	171.	179.
163.	228.	224.	255.	236.	246.	249.	206.	151.	179.	155.	115.
90.	191.	438.	429.	474.	472.	402.	431.	451.	339.	234.	144.
89.	120.	152.	202.	390.	362.	342.	388.	400.	294.	217.	133.
94.	222.	220.	302.	292.	264.	340.	324.	282.	471.	271.	175.
159.	213.	195.	244.	279.	258.	255.	270.	187.	122.	85.	52.
36.	112.	201.	332.	426.	366.	348.	382.	374.	293.	196.	128.
103.	268.	341.	710.	542.	362.	327.	297.	212.	151.	124.	98.
174.	303.	358.	343.	435.	393.	400.	433.	322.	210.	137.	91.
120.	118.	208.	291.	372.	292.	341.	376.	302.	265.	217.	146.
128.	157.	380.	489.	455.	396.	413.	395.	272.	175.	108.	64.
43.	64.	95.	166.	320.	406.	286.	226.	178.	139.	90.	64.
124.	214.	269.	286.	269.	405.	424.	404.	324.	230.	183.	149.
136.	174.	224.	187.	207.	343.	375.	375.	378.	299.	234.	213.
178.	193.	410.	448.	570.	380.	345.	406.	451.	320.	210.	148.
148.	270.	404.	508.	389.	383.	262.	330.	517.	403.	286.	191.
126.	222.	257.	257.	333.	374.	404.	446.	434.	300.	212.	171.
125.	126.	144.	189.	225.	303.	325.	329.	297.	225.	154.	104.
83.	202.	264.	373.	606.	486.	442.	457.	436.	307.	207.	123.
117.	216.	273.	324.	429.	351.	326.	352.	339.	335.	315.	225.
172.	207.	253.	432.	608.	481.	444.	447.	396.	341.	254.	169.
111.	216.	447.	490.	635.	459.	379.	433.	401.	292.	190.	120.
83.	90.	351.	375.	360.	308.	324.	369.	301.	321.	262.	192.
122.	169.	217.	212.	306.	299.	323.	316.	277.	213.	167.	139.
152.	216.	262.	337.	304.	306.	309.	381.	385.	364.	268.	175.
104.	76.	303.	418.	314.	253.	280.	280.	250.	231.	146.	92.
72.	162.	516.	450.	416.	394.	433.	442.	424.	282.	176.	106.
64.	116.	241.	549.	540.	391.	438.	454.	347.	210.	120.	71.
42.	107.	214.	192.	516.	606.	476.	478.	538.	352.	242.	186.

Expected Annual Flow-Increase EFI_ [in]

	BASIN A	BASIN B
7-->	-------	-------

```
          > 10.       > 10.


       Number of Acres Treated  ACT_ [acres]
       -----------------------------------
            BASIN A     BASIN B
8-->        --------    --------
          > 650000.   > 550000.


    Monthly Distribution of Annual Flow-Increases  PIF_[%]
    ------------------------------------------------------
              Basin A       Basin B
9-->          -------       -------
               .00<          .00<
               .05           .05
               .10           .10
               .15           .15
               .20           .20
               .20           .20
               .15           .15
               .10           .10
               .05           .05
               .00           .00
               .00           .00
               .00<          .00<

ENDFILE(UNIT=1)
```

APPENDIX II

< SYSTEM.DAT >

Input File for Module-2

```
***************************************
*      S Y S T E M. D A T             *
***************************************
```

	RESERVOIR SUBSYSTEMS				
	RESERVOIR A		RESERVOIR C		Units
1--> Maximum Storage	> 8500.		> 14000.		[MCM]
2--> Minimum Storage	> 2300.		> 6000.		[MCM]
3--> Initial Storage	> 4000.		> 9000.		[MCM]
4--> Final Storage	> 4000.		> 9000.		[MCM]
5--> Storage at Spillway Crest Level	> 5000.		> 9100.		[MCM]
6--> Coeff. of Storage/ /Outflow function	> 000.	> 000.	> 000.	> 000.	[ ]
7--> Coeff. of Storage/ /Elevation function	> 1.861	> .435	> 3.03	> .385	[MCM/m]
8--> Coefficient of Area/ /Storage function	> 1.278	> .560	> 0.298	> .731	[KM2/MCM]
9--> Evaporation Rate	>		>		[m/mo]
month 1 ...	0.20		0.20		
month 2 ...	0.19		0.20		
month 3 ...	0.17		0.17		
month 4 ...	0.09		0.10		
month 5 ...	0.08		0.07		
month 6 ...	0.10		0.07		
month 7 ...	0.08		0.06		
month 8 ...	0.08		0.07		
month 9 ...	0.09		0.09		
month 10 ...	0.15		0.15		
month 11 ...	0.16		0.19		
month 12 ...	0.25		0.25		
10-> Max.Turbine Release	> 2200.		> 3500.		[MCM/mo]
11-> Generation Efficiency	> 0.87		> 0.87		[%]

	WATER DEMAND SUBSYSTEMS ZONE A	ZONE C	
12-> Minimum Required Water-Supply			[MCM/mo]
month 1 ...	> 0.0	> 0.0	
month 2 ...	0.0	0.0	
month 3 ...	0.0	0.0	
month 4 ...	0.0	0.0	
month 5 ...	0.0	0.0	
month 6 ...	0.0	0.0	
month 7 ...	0.0	0.0	
month 8 ...	0.0	0.0	
month 9 ...	0.0	0.0	
month 10 ...	0.0	0.0	
month 11 ...	0.0	0.0	
month 12 ...	0.0	0.0	

	RECREATION AREAS REACH A	REACH C	
13-> Minimun Required Recreation Flows			[MCM/mo]
month 1 ...	> 200.	> 500.	
month 2 ...	200.	500.	
month 3 ...	150.	400.	
month 4 ...	150.	400.	
month 5 ...	100.	300.	
month 6 ...	100.	300.	
month 7 ...	100.	300.	
month 8 ...	100.	300.	
month 9 ...	150.	400.	
month 10 ...	150.	400.	
month 11 ...	200.	500.	
month 12 ...	200.	500.	

ENDFILE(UNIT=2)

APPENDIX III

< ECONOM.DAT >

Input File for Module-3

```
************************************
*        E C O N O M. D A T        *
************************************
```

	HYDROPOWER SUBSYSTEMS				
	POWERPLANT A		POWERPLANT C		Units
	a1A ↓	b1A ↓	a1C ↓	b1C ↓	
1--> Energy Rate Function	> 97.0	> 0.022	> 163.	> 0.012	[MWh/MCM]
	a2A ↓	b2A ↓	a2C ↓	b2C ↓	
2--> Energy Demand Curve	> 44.5	> -0.016	> 44.0	> -0.009	[$/MWh]
3--> Annual Energy Price	> 2.00				[c/KWh]
4--> Monthly Energy Price	>		>		[c/KWh]
month 1 ...	2.00		2.00		
month 2 ...	2.00		2.00		
month 3 ...	2.00		2.00		
month 4 ...	2.00		2.00		
month 5 ...	2.00		2.00		
month 6 ...	2.00		2.00		
month 7 ...	2.00		2.00		
month 8 ...	2.00		2.00		
month 9 ...	2.00		2.00		
month 10 ...	2.00		2.00		
month 11 ...	2.00		2.00		
month 12 ...	2.00		2.00		

	WATER DEMAND SUBSYSTEMS				
	ZONE A		ZONE C		
5--> Water-Supply Demand Curve					[$/MCM]
month 1 ...	> 25000.	> -0.001	> 20000.	> -0.001	
month 2 ...	25000.	-0.001	20000.	-0.001	
month 3 ...	50000.	-0.003	40000.	-0.003	
month 4 ...	50000.	-0.003	40000.	-0.003	
month 5 ...	80000.	-0.015	60000.	-0.015	
month 6 ...	80000.	-0.015	60000.	-0.015	
month 7 ...	80000.	-0.015	60000.	-0.015	
month 8 ...	80000.	-0.015	60000.	-0.015	
month 9 ...	50000.	-0.003	40000.	-0.003	
month 10 ...	50000.	-0.003	40000.	-0.003	
month 11 ...	25000.	-0.001	20000.	-0.001	
month 12 ...	25000.	-0.001	20000.	-0.001	

ENDFILE(UNIT=2)

APPENDIX IV

DERIVATIVES OF THE OBJECTIVE FUNCTION

Appendix IV provides the analytical expressions for all partial derivatives utilized in the QP formulation. The following derivations consider the general case illustrated in Chapter 2, Figure 2.5, with a reservoir receiving releases from an upstream powerplant, and in turns, the releases from the referred powerplant affecting a downstream reservoir.

OBJECTIVE FUNCTIONS

The objective function (O.F.) that computes the return stemming from hydropower, Eq.(2.9), and from off-stream water supply, Eq.(2.12), at any time period i, are given respectively by :

O.F. for Hydropower

$$B_i^{POW}[\$] = \eta \, \frac{p_i}{P} \left\{ T_i \left[a_1 a_2 + b_1 a_2 S_i^{\circ} + \frac{b_1 a_2}{2} INF_i - \frac{b_1 a_2}{2} LOS_i \right] + T_i^2 \left[\frac{a_1 b_2}{2} - \frac{b_1 a_2}{4} + \frac{b_1 b_2}{2} S_i^{\circ} + \frac{b_1 b_2}{4} INF_i - \frac{b_1 b_2}{4} LOS_i \right] - T_i^3 \left[\frac{b_1 b_2}{6} \right] \right\} \qquad for\ i = 1,2,\ldots,N_p \qquad \text{(IV.1)}$$

O.F. for Water-Supply

$$B_i^{WS} = \frac{a_3}{b_3} \left\{ \exp\left(b_3 D_i\right) - 1 \right\} \qquad for\ i = 1,2,\ldots,N_p \qquad \text{(IV.2)}$$

where all symbols are as defined in Chapter 2. Before proceeding to compute the partial derivatives via calculus, it is convenient to perform some formula manipulation to write Eq.(IV.1) in its most explicit way possible, in order to clearly differentiate all the variables that are subject to differentiation. In Eq.(IV.1), the reservoir storage at the beginning of period i , S_i° , is a function of the history of inflows and outflows from the reservoir up to that period, that is:

$$S_i^{\circ} = S^{\circ} + \sum_{k=1}^{i-1} (INF_k) - \sum_{k=1}^{i-1} (OUT_k) \qquad \text{(IV.3)}$$

where in turns, inflows and outflows at any given time period k are given by,

$$INF_k = Q_k + T_k^* + L_k^* - D_k^* \tag{IV.4a}$$

$$OUT_k = T_k + L_k + E_k \tag{IV.4b}$$

Then substituting Eqs.(IV.4a) and (IV.4b) into Eq.(IV.3), the latter becomes,

$$S_i^{\circ} = S^{\circ} + \sum_{k=1}^{i-1} (Q_k + T_k^* + L_k^* - D_k^*) - \sum_{k=1}^{i-1} (T_k + L_k + E_k) \tag{IV.5}$$

Furthermore, the parameters $\{a_1, b_1, a_2, b_2, a_3, b_3, S^{\circ}\}$ in Eqs.(IV.1) and (IV.2) are grouped in the following manner,

$$\begin{array}{lll} C1 = a_1 a_2 & C5 = 1/4\, b_1 b_2 & C9 = 1/2\, b_1 b_2 S^{\circ} \\ C2 = 1/2\, b_1 a_2 & C6 = 1/2\, b_1 b_2 & C10 = a_3 / b_3 \\ C3 = b_1 a_2 & C7 = 1/6\, b_1 b_2 & C11 = b_3 \\ C4 = 1/2\, a_1 b_2 - 1/4\, b_1 a_2 & C8 = b_1 a_2 S^{\circ} & C12 = a_3 \end{array} \tag{IV.6}$$

Finally, after substituting Eq.(IV.5) into (IV.1), and then, by grouping the constant coefficients according to (IV.6), the final form for the objective functions become:

O.F. for Hydropower

$$\begin{aligned} B_i^{POW} = \eta \frac{p_i}{P} \Big\{ & T_i \Big[C1 + C8 + C3 \sum_{k=1}^{i-1} (Q_k + T_k^* + L_k^* - D_k^*) - C3 \sum_{k=1}^{i-1} (T_k + L_k + E_k) \\ & \quad + C2\,(Q_i + T_i^* + L_i^* - D_i^*) - C2\,(L_i + E_i) \Big] \\ & + T_i^2 \Big[C4 + C9 + C6 \sum_{k=1}^{i-1} (Q_k + T_k^* + L_k^* - D_k^*) - C6 \sum_{k=1}^{i-1} (T_k + L_k + E_k) \\ & \quad + C5\,(Q_i + T_i^* + L_i^* - D_i^*) - C5\,(L_i + E_i) \Big] \\ & - T_i^3 \Big[C7 \Big] \Big\} \qquad \textit{for } i = 1,2,\ldots,N_p \end{aligned} \tag{IV.7}$$

O.F. for Water-Supply

$$B_i^{WS} = C10_i * \Big\{ exp\big(C11_i\, D_i\big) - 1 \Big\} \qquad \textit{for } i = 1,2,\ldots,N_p \tag{IV.8}$$

Notice that in the most general case, the coefficients C10, C11, and C12 can vary in time.

FIRST PARTIAL DERIVATIVES (FPD)

As indicated in Chapter 2, spillages and evaporation losses are considered as constant terms during each sequence of SQP, hence, L and E will not be treated as variables during the differentiation. Recall that only T and D are the control variables during the optimization process. Also remenber the difference between T_i , T_i^u and T_i^d. The three variables represent turbine releases during the same time period i , however, T_i corresponds to the reservoir under consideration, T_i^u represents a release from the upstream powerplant, and T_i^d represents a release from the powerplant located downstream.

Another important aspect to take into consideration when deriving the partial derivatives is the dependence of the parameters η, C1, C2, ..., C12 with time and space. All of them are site dependent, that is, each subsystem (reservoir) has its own set of parameters. Therefore, the same notation for superscripts [^{u}, ^{d}] utilized to differentiate variables was also used to associate a given parameter to the subsystem it belongs to. However, not all the parameters change with time. Only those associated with the objective function for water-supply, C10, C11, and C12 are period dependent, that is, they may change from period to period. For hydropower, it is the ratio p_i/P the one that manifests possible changes of the energy demand curve from period to period.

When computing the partial derivatives we should look at the global objective function (G.O.F.) of the system , made up by the single objective functions (O.F.) for hydropower and water-supply, for all sites and for all time periods.

FPD of the G.O.F. with respect to T_i

The variable T_i shows up clearly on Eq.(IV.7), computing hydropower return during period i. On the contrary, there isn't any T_i as part of Eq.(IV-8). However, when considering the overall O.F., we can see that a given T_i appears in the global objective function playing three different roles, they are: a) as a turbine release during period i from the reservoir under analysis, b) as an outflow from the same reservoir when considering time periods posterior to i , and, c) as part of the inflow term into the downstream reservoir during period i and all periods following in time. Then the general expression for the partial derivative is given by:

$$\frac{\partial B^{POW}}{\partial T_i} = \eta \frac{p_i}{P} \Big\{ \Big[C1 + C8 + C2\,(Q_i + T_i^u + L_i^u - D_i^u) - C2\,(L_i + E_i)$$
$$+ C3 \sum_{k=1}^{i-1} (Q_k + T_k^u + L_k^u - D_k^u) - C3 \sum_{k=1}^{i-1} (T_k + L_k + E_k) \Big]$$
$$+ 2\,T_i \Big[C4 + C9 + C5\,(Q_i + T_i^u + L_i^u - D_i^u) - C5\,(L_i + E_i)$$
$$+ C6 \sum_{k=1}^{i-1} (Q_k + T_k^u + L_k^u - D_k^u) - C6 \sum_{k=1}^{i-1} (T_k + L_k + E_k) \Big]$$
$$- 3\,T_i^2 \Big[C7 \Big]$$
$$- \Big[C3 \sum_{j=i+1}^{N_p} T_j + C6 \sum_{j=i+1}^{N_p} (T_j)^2 \Big] \Big\}$$
$$+ \eta^d \frac{p_i{}^d}{P} \Big\{ \Big[C2^d\, T_i^d + C3^d \sum_{j=i+1}^{N_p} T_j^d + C5^d\, (T_i^d)^2 + C6^d \sum_{j=i+1}^{N_p} (T_j^d)^2 \Big] \Big\} \qquad \text{(IV.9)}$$

FPD of the G.O.F. with respect to D_i

The variable D_i is part of the overall objective function under the following two roles: a) as part of Eq.(IV.8) computing the return from off-stream water supply, and, b) as part of the terms that account for inflows in Eq.(IV.7), for time periods i and preceding. The expression for the partial derivative is given by:

$$\frac{\partial B^{WS}}{\partial D_i} = + C12_i \exp\Big(C11_i\, D_i \Big)$$
$$- \eta^d \frac{p_i{}^d}{P} \Big[C2^d\, T_i^d + C5^d\, (T_i^d)^2 + C3^d \sum_{j=i+1}^{N_p} T_j^d + C6^d \sum_{j=i+1}^{N_p} (T_j^d)^2 \Big] \qquad \text{(IV.10)}$$

SECOND PARTIAL DERIVATIVES (SPD)

Once the expressions for the first partial derivatives were obtained, the expressions for the second partial derivatives are readily derived from those,

SPD of the G.O.F. with respect to T_i

$$\frac{\partial^2 B^{POW}}{\partial T_i \partial T_i} = \eta \frac{p_i}{P} \left\{ 2 \left[C4 + C9 + C5 (Q_i + T_i^{*} + L_i^{*} - D_i^{*}) - C5 (L_i + E_i) + C6 \sum_{k=1}^{i-1} (Q_k + T_k^{*} + L_k^{*} - D_k^{*}) - C6 \sum_{k=1}^{i-1} (T_k + L_k + E_k) \right] - 6\, T_i \left[C7 \right] \right\} \tag{IV.11}$$

SPD of the G.O.F. with respect to D_i

$$\frac{\partial^2 B^{WS}}{\partial D_i \partial D_i} = C11_i\, C12_i\, \exp\left(C11_i\, D_i \right) \tag{IV.12}$$

SECOND CROSS PARTIAL DERIVATIVES (CPD)

Second-cross partial derivatives should be carefully derived from Eqs.(IV.9) and (IV.10). There are two groups of second-cross partial derivatives. One group is the set of expressions derived from Eq.(IV.9), that is, the second cross derivatives after taking the first derivative with respect to T_i, resulting in Eq.(IV.13), and the second group of cross derivatives, derived from Eq.(IV.10), after the first derivative was taken with respect to D_i, resulting in Eq.(IV.14). The following notation for time running indexes is critical for the understanding of the expressions given next, that is:

index ... i	denotes the present time period
index ... k	denotes any time period prior to i, that is → k = 1, 2, 3, ..., (i-2), (i-1)
index ... j	denotes any time period a posteriori of i, that is → j = (i+1), (i+2), ..., N_p

CPD after FPD with respect to T_i (IV.13)

$$\frac{\partial^2 B}{\partial T_i \partial T_k^u} = \eta \, \frac{p_i}{P} \, (C3 + 2\, C6\, T_i)$$

$$\frac{\partial^2 B}{\partial T_i \partial T_i^u} = \eta \, \frac{p_i}{P} \, (C2 + 2\, C5\, T_i)$$

$$\frac{\partial^2 B}{\partial T_i \partial T_j^u} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial D_k^u} = \eta \, \frac{p_i}{P} \, (-C3 - 2\, C6\, T_i)$$

$$\frac{\partial^2 B}{\partial T_i \partial D_i^u} = \eta \, \frac{p_i}{P} \, (-C2 - 2\, C5\, T_i)$$

$$\frac{\partial^2 B}{\partial T_i \partial D_j^u} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial T_k} = \eta \, \frac{p_i}{P} \, (-C3 - 2\, C6\ T_i)$$

$$\frac{\partial^2 B}{\partial T_i \partial T_i} = \text{Eq.}\,(IV.11)$$

$$\frac{\partial^2 B}{\partial T_i \partial T_j} = \eta \, \frac{p_i}{P} \, (-C3 - 2\, C6\ T_j)$$

$$\frac{\partial^2 B}{\partial T_i \partial D_k} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial D_i} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial D_j} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial T_k^d} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial T_i^d} = \eta^d \, \frac{p_i}{P}^d \, (C2^d + 2\, C5^d\, T_i^d)$$

$$\frac{\partial^2 B}{\partial T_i \partial T_j^d} = \eta^d \, \frac{p_i}{P}^d \, (C3^d + 2\, C6^d\, T_j^d)$$

$$\frac{\partial^2 B}{\partial T_i \partial D_k^d} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial D_i^d} = 0$$

$$\frac{\partial^2 B}{\partial T_i \partial D_j^d} = 0$$

CPD after FPD with respect to D_i (IV.14)

$$\frac{\partial^2 B}{\partial D_i \partial T_k^u} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_i^u} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_j^u} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_k^u} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_i^u} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_j^u} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_k} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_i} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_j} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_k} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_i} = \text{Eq.}\,(IV.12)$$

$$\frac{\partial^2 B}{\partial D_i \partial D_j} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_k^d} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial T_i^d} = \eta^d \, \frac{p_i}{P}^d \, (-C2^d - 2\, C5^d\, T_i^d)$$

$$\frac{\partial^2 B}{\partial D_i \partial T_j^d} = \eta^d \, \frac{p_i}{P}^d \, (-C3^d - 2\, C6^d\, T_j^d)$$

$$\frac{\partial^2 B}{\partial D_i \partial D_k^d} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_i^d} = 0$$

$$\frac{\partial^2 B}{\partial D_i \partial D_j^d} = 0$$

APPENDIX V

STREAMFLOW CHARACTERISTICS IN CENTRAL AND WESTERN UNITED STATES

INTRODUCTION

This appendix presents a characterization of normal flow regimes in the central and western portions of the U.S. Flow characteristics are derived from streamflow records that were collected from a network of 71 streamflow gaging stations to the west of the Mississippi River. The gaging stations selection criterion was primarily based on streams with unregulated flows and with none or negligible diversions. Stations with the largest drainage areas were the most desirable. However, there is a tradeoff between the size of the drainage basin and the possibility of dealing with flow unaffected by human activities. The selected gaging stations provide records suitable to define natural streamflow characteristics throughout the area of interest.

BACKGROUND

Although extensive work has been done by the USGS in the area of surface hydrology, it does not exist, to the best of our knowledge, a comprehensive study encompassing the characterization of average flow regimes for the whole United States. Certainly, studies of this type have been done in a local basis, for instance, at a county level or at the most for a single state. There is an understandable reason for having focused most of the regionalization studies done by the USGS and other research institutions in extreme flow conditions, without paying much attention to mean-flow conditions. Low-flow and peak-flow events are the ones calling the attention of most water-resources managers, since those are the ones imposing the most adverse effects to any water use.

Likewise, most of the work presented in the literature concerning regional analysis of streamflow characteristics, concentrate on the

development and comparison of techniques applicable for hydrologic regionalization. As an example, we can mention:

- Discriminant analysis to flood data,
- Factor analysis for grouping basins with similar basin characteristics,
- Cluster analysis on the basis of drainage basin characteristics (area, slope, etc.),
- Cluster analysis on the basis of the discharge data (coefficient of variation, ratio Q/area, etc.).

METHODOLOGY USED

This study intends to identify areas with similar flow regimes for the whole central and western U.S. regions, regardless of political boundaries. The first attempted approach to perform the regional identification of areas with comparable flow regimes was to utilize the well known division of principal river basins. However, it was found that the highly variable hydrological conditions within a single water-resources region made the attempt impossible. It is interesting, but in fact should not be surprising that, when dividing the country in physiographical provinces, the identification of regions with identical flow regimes can be readily done.

The pioneering work in this area goes back to Fenneman [1931], when he presented his book analyzing the land forms of the western U.S. The country was divided into natural physical units called provinces. Each province is in general characterized by a unique climatic factor, as well as physiographic and geological conditions that control patterns of stream runoff. Very useful for this study has been the National Water Summary publication prepared by the USGS [1985]. The

hydrologic information provided by this water supply paper, in a state by state basis, has permitted to achieve the objective.

Because of the variety of geographic and climatic conditions, surface water resources differ considerably among all provinces. The climatic, geologic and topographic differences among the provinces greatly affect the distribution and variation in precipitation and runoff throughout. That is why rivers that flow through two or more physiographic provinces are likely to have their runoff and flow characteristics changing abruptly. This is certainly the case when the analysis is interpreted under the river-basin viewpoint.

In this study, the identification of regional differences in runoff patterns was done by visual inspection, grouping stations depicting similar histograms form. This approach, though simple in conception, provides a useful and quick answer to the requirements of the present study.

PRESENTATION OF FINDINGS

This section presents the results from the analysis. The report provides the following information:

1. List of physiographic regions west of the Mississippi River (physical boundary for our study), including the list of the selected gaging stations.
2. Map 1 depicting the location of the selected gaging stations analyzed at each physiographic province. Stations were spread out within each province in order to cover the maximum area with a reasonable number of stations.
3. Brief description of the general setting at each physiographic province, highlighting the main climatic,

geographic and hydrologic factors that contribute to shape runoff histograms.

4. Statistical analysis of mean monthly flows series at each station. The analysis serves two different aspects:

   (a) Seasonal (monthly) Variability. Including the following statistics of the periodic time series: monthly means $\bar{x}_i$, standard deviation of monthly means σ_i, and the lag-1 serial correlation structure ρ_{1i}, i.e. the correlation of the flow at month (i) with the flow at the previous month (i-1).

   (b) Interannual Variability. Including statistics of the mean annual flow series like: mean annual flow $\bar{x}_a$, standard deviation of mean annual flow, σ_a; and coefficient of variation of the annual series C_v.

   Listings with the time series for each station and resulting statistics are provided.

5. Bar graphs showing streamflow patterns of mean monthly flows at selected sites.

6. Map 2 showing typical regional differences in runoff patterns associated with different physiographic provinces.

LIST OF PHYSIOGRAPHIC REGIONS AND GAGING STATIONS

1. PACIFIC BORDER:

   NASELLE RIVER NEAR NASELLE - (12010000)
   UMPQUA RIVER NEAR ELKTON, OREG. - (14321000)
   WILSON RIVER NEAR TILLAMOOK, OREG. - (14301500)
   SMITH RIVER NEAR CRESCENT CITY, CALIF. - (11532500)
   NAVARRO RIVER NEAR NAVARRO, CALIF. - (11468000)
   SAN LORENZO R AT BIG TREES CALIF - (11160500)
   ARROYO SECO NR PASADENA CALIF - (11098000)

2. CASCADE SIERRA MOUNTAINS:

KERN RIVER AT KERNVILLE CALIF - (11187000)
MCCLOUD RIVER NR MCCLOUD CALIF - (11367500)
MERCED R AT HAPPY ISLES BRIDGE NR YOSEMITE CALIF - (11264500)
ROGUE RIVER ABOVE PROSPECT, OREG. - (14328000)
MCKENZIE R NR VIDA OREG - (14162500)
KLICKITAT RIVER NEAR PITT, WASH. - (14113000)
THUNDER CREEK NR. NEWHALEM, WASH. - (12175500)

3. COLUMBIA PLATEAUS:

SILVIES RIVER NEAR BURNS,OREG. - (10393500)
JOHN DAY R AT MCDONALD FERRY,OREG. - (14048000)
PALOUSE RIVER AT HOOPER, WASH. (13351000)
BRUNEAU RIVER NR HOT SPRINGS ID - (13168500)

4. NORTHERN-ROCKY MOUNTAINS :

LOCHSA RIVER NR LOWELL ID - (13337000)
COLVILLE RIVER AT KETTLE FALLS, WASH. - (12409000)
BIG HOLE RIVER NEAR MELROSE, MT. - (06025500)
CLARK FORK AT ST. REGIS, MT. - (12354500)

5. CENTRAL-ROCKY MOUNTAINS :

BIGHORN R AT KANE WYO - (06279500)
WEBER RIVER NEAR OAKLEY, UTAH - (10128500)
SMITHS FORK NEAR BORDER, WY - (10032000)

6. SOUTHERN-ROCKY MOUNTAINS:

NORTH PLATTE RIVER NEAR NORTHGATE, CO. - (06620000)
RIO GRANDE NEAR LOBATOS, CO. - (08251500)
PECOS R NR PECOS, NM - (08378500)

7. WYOMING BASINS:

GREEN RIVER AT WARREN BRIDGE, NEAR DANIEL, WY - (09188500)
SWEETWATER RIVER NEAR ALCOVA, WYO. - (06639000)

8. COLORADO PLATEAUS:

ZUNI RIVER ABV BLACK ROCK RESERVOIR, N M - (09386950)
GUNNISON RIVER NEAR GRAND JUNCTION, CO. - (09152500)
COLORADO RIVER NEAR CISCO UTAH - (09180500)
ANIMAS RIVER AT DURANGO, CO. - (09361500)
COLORADO R AT LEES FERRY, AZ. - (09380000)

9. BASIN AND RANGE:

BILL WILLIAMS RIVER BLW ALAMO DAM, AZ. - (09426000)
SALT CREEK NR MECCA CALIF - (10254050)
SAN PEDRO RIVER AT PALOMINAS, ARIZ. - (09470500)
GILA RIVER NEAR GILA, NM - (09430500)
SANTA CRUZ RIVER AT TUCSON, ARIZ. (09482500)
BEAVER RIV NR BEAVER UTAH - (10234500)
HUMBOLDT R AT PALISADE, NV - (10322500)
LIMPIA CREEK ABOVE FORT DAVIS TX(DISC) - (08431700)
S TWIN R NR ROUND MOUNTAIN, NV - (10249300)

10. CENTRAL LOWLAND:

BRAZOS RIVER NR SOUTH BEND, TX - (08088000)
CIMARRON RIVER AT PERKINS, OK - (07161000)
BIG NEMAHA RIVER AT FALLS CITY, NEBR. - (06815000)
L ARKANSAS R AT VALLEY CENTER, KS - (07144200)
SHELL ROCK RIVER AT SHELL ROCK, IOWA - (05462000)
MINNESOTA RIVER NEAR JORDAN, MN - (05330000)
SKUNK RIVER AT AUGUSTA, IOWA - (05474000)
BIG SIOUX R NEAR DELL RAPIDS SD - (06481000)
RED RIVER OF THE NORTH AT HALSTAD, MN - (0506450)

11. OZARK PLATEAUS:

FOURCHE MALINE RIVER NR RED OAK, OK - (07247500)
MULBERRY RIVER NEAR MULBERRY, ARK. - (07252000)
SPRING RIVER AT IMBODEN, ARK. - (07069500)
GASCONADE RIVER AT JEROME MO - (06933500)

12. COASTAL PLAIN:

BAYOU BARTHOLOMEW NEAR MCGEHEE, ARK - (07364150)
NECHES RIVER NEAR ROCKLAND, TEX. - (08033500)
CALCASIEU R NR OBERLIN, LA - (08013500)

13. GREAT PLAINS:

LLANO RIVER NR JUNCTION, TX - (08150000)
CANADIAN RIVER NR AMARILLO, TX - (07227500)
MILK RIVER AT NASHUA MT - (06174500)
SMOKY HILL R AT ELKADER, KS - (06860000)
PURGATOIRE RIVER NEAR LAS ANIMAS, CO. - (071285001)
LITTLE MISSOURI RIVER NR WATFORD CITY, ND - (06337000)
DELAWARE RIVER NR RED BLUFF, N M - (08408500)
CANNONBALL RIVER AT BREIEN, ND - (06354000)
L POWDER RIVER AB DRY C NR WESTON, WY - (06324970)
POWDER RIVER NEAR LOCATE, MT. - (06326500)
KEYA PAHA R AT WEWELA SD - (06464500)

PHYSIOGRAPHIC PROVINCES

<u>Pacific Border</u>

This province extends all along the Pacific Ocean coast, from the state of Washington to Oregon and California. In all this region prevails western winds from the Pacific Ocean. It has a marine climate, with cool, wet winters and warm, relatively dry summers. The largest portion of the precipitation falls in this area, mainly concentrated in the fall and winter. The rainfall-runoff regime predominates in this province. Runoff is closely related to precipitation, and because of the relatively low elevation, the heavy winter precipitation falls as rain and runs off quickly. In the northern portion of the province, Washington and oregon, the distribution of the runoff is with the maximum runoff occurring in November through March, and the period of low-flow last all summer. As we move south to the California coast, the general runoff pattern remains unchanged, with a moderate shift of the histogram peak toward later in the winter.

<u>Cascade Sierra Mountains</u>

This North-South trending mountainous region extends from Washington to Oregon and California, practically parallel to the Pacific Border. The Cascades divide the three states into the west side with a marine climate and the east side with a continental climate. The eastern side of the mountains receives much less precipitation than the west side. Storms that reach the western side of the Cascade Range release much of the remaining moisture. Most of the precipitation is in the form of snow. Heavy snowpacks and glaciers are major sources of water for many rivers. This natural reservoir of

water does not runoff until late spring or early summer. The period of low flow last all winter.

Columbia Plateaus

This is a province with varied topography, with mountain ranges and plains in southern Idaho and eastern Oregon, and uplands in western Idaho and southeastern Washington. Most precipitation (approximately 2/3) in this province falls as snow in the fall and winter, from October through March, resulting in the typical snowmelt-runoff regime.

Rocky Mountains

The Rocky Mountains province is divided in three subprovinces, they are: the Northern Rocky Mountains, the Middle Rocky Mountains and the Southern Rocky Mountains. Even though each subregion presents singularities as far as its topographic conditions, for the purpose of this study they will be considered as a unity, since runoff patterns do not differ significantly from one subprovince to another. Most of the larger streams have their headwaters in this region. As expected, most precipitation in the mountains is in the form of snow during the winter season, November through February. A large percentage of the runoff is a result of snowmelt, which does not occur until the warmer temperatures of spring and early summer. By July the snow has melted and the streamflow becomes low. Snowmelt runoff in the Rocky Mountains is delayed one or two months with respect to other provinces with low terrain. As we move south in the Rocky Mountain range, in the northern portion of New Mexico, much of the runoff from the mountains occurs during concurrent snowmelt and rainfall in spring and summer.

Wyoming Basin

This province extends mainly in the south-central portion of the state of Wyoming. This plateau is interposed between the Central and

Southern Rocky Mountains. Most precipitation in this intermontane basins and on the plains is rain from summer thunderstorms. The histogram peak takes place during May and June, with the least precipitation from December through February. Much of the precipitation evaporates or is transpired by vegetation.

Colorado Plateaus

This region presents mountain ranges, high plateaus, and broad basins, which locally have been deeply incised primarily by the Colorado River and its tributaries. Most runoff results from melting of the large snowpack in the mountains, from April through July.

Basin and Range

This is a large province which takes practically the whole state of Nevada and extends along the border with Mexico through southern California, Arizona, New Mexico and a small portion of Texas. In order to accommodate two different flow patterns it was necessary to divide the province into two subprovinces. The first one, constituted mainly by the state of Nevada and west Utah, is characterized by isolated, roughly north-south parallel mountain ranges, and broad intervening relatively flat valleys. The orographic effect of the mountains induces precipitation from storm systems generally in the form of snow. In the nonmountainous areas, virtually all the precipitation evaporates. Spring and summer snowmelt supplies most of Nevada and Utah's streamflow, mostly during May and June. The second subprovince, in the southern portion, is characterized by lowlands with broad alluvium-filled valleys bounded by steeply rising mountain ranges (Arizona). The summer season, particularly during July and August, is the wettest, and precipitation occurs as intensive thunderstorms of

short duration. The second rainy season is December through mid-March. May and June are the driest months.

Central Lowland

This large province extends north-south and is mainly characterized by level to rolling glaciated plains. The northern portion is gentle sloping with poorly defined drainage patterns. The central part presents extensively farmed lowland with numerous, wide, flat valleys eroded by meandering streams. Fortunately, a large percentage of annual runoff occurs during the growing season, April through October. Particularly in the northern states, North Dakota, South Dakota, and Minnesota, runoff occurs as a result of snowmelt (moisture stored during winter) and rainfall in the spring and early summer. In the central and southern portion of the province most of the streamflow occurs from direct runoff from rainfall during spring or early summer, May, June or July. The period of least discharge usually occurs in December or January.

Great Plains

This is the largest physiographic province in the U.S. It extends north-south from the Canadian border to the Mexican border. As expected for such a large region, it was necessary to subdivide the province in order to define subareas with approximately equal patterns of stream runoff. The northern subarea, with rolling prairie land, presents less snowfall than in the Rocky Mountains. Moisture stored during the winter as snow generally begins to melt and runoff in March. Snowmelt runoff in the Great Plains occurs one to two months earlier than snowmelt runoff in the Rocky Mountains. Runoff occurs as a result of snowmelt and rainfall in the spring and early summer. The greatest

runoff occurring in the west-boundary from foothill streams during the adjacent Rocky Mountains, and decreasing towards the east.

The central subarea, with dissected plains, high plains and rolling hills, presents average runoff closely related to average monthly precipitation. Most runoff is from snowmelt in spring and mainly from thunderstorms during spring and summer.

The southern subarea has an arid to semiarid climate, where smaller streams are ephermeral, and with treeless areas and prairies in western Texas. Runoff results from short duration, intense rainstorms. Much of the rainfall falls during May through September. Many months have little or no precipitation.

Coastal Plain

The Coastal Plain province is a rolling to hilly rangeland that rises from sea level (below sea level in southern-Louisiana) in the Gulf of Mexico to about 500 feet near the base of the Balcones Escarpment. Runoff primarily reflects precipitation patterns in the region. It varies seasonally and areally depending on precipitation patterns. The greatest runoff typically occurs from january through May.

Ozark Plateaus

The Ozark Region, constituted by portions of Missouri, Arkansas and Oklahoma, is a wooded, hilly region, as high as 2700 feet above sea level in Arkansas. Major Ozark Plateaus streams are sustained by thousands of springs in the region, tending to have sustained flows during dry seasons. Because maximum precipitation occurs mostly in May, spring is the wettest season. Due to the incidence of hot, dry weather in late summer and early fall and excessive evapotranspiration rates, runoff usually is at a minimum in August and September.

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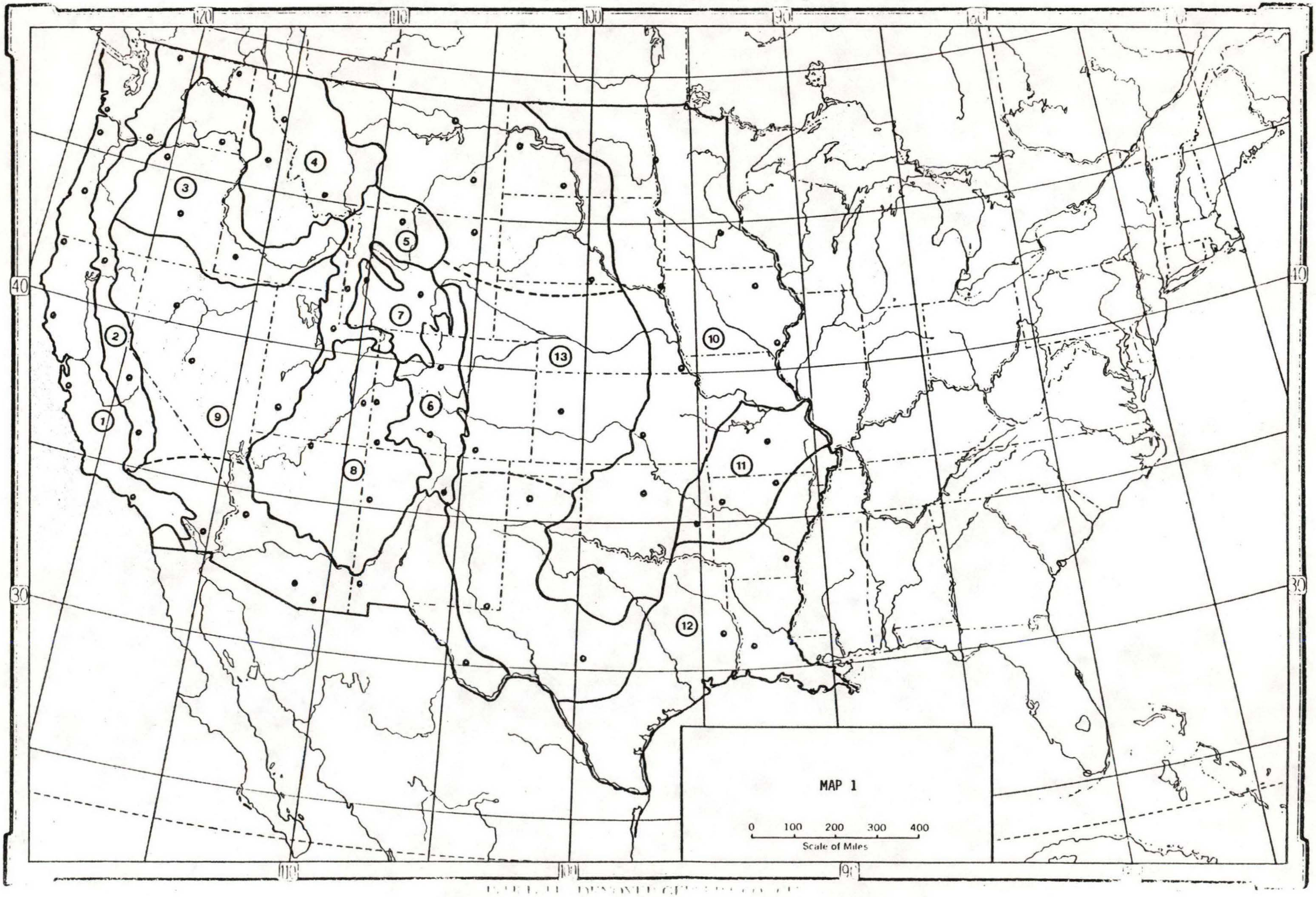

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